

SATURATED AND UNSATURATED HYDRAULIC PROPERTIES CHARACTERIZATION AT MINE FACILITIES: ARE WE DOING IT RIGHT¹?

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The accurate determination of saturated and unsaturated hydraulic properties of mine waste and cover material is critical for predicting long-term drainage behavior and closure performance. The rock fragments typically found in mine waste and borrow materials complicate laboratory hydraulic property measurements. Many hydraulic testing laboratories address this issue by removing all material greater than 4.75 mm in diameter, repacking the remaining fine-earth sample in small diameter cores, and “correcting” the resulting measurements for the gravel content using published correction factors.

In order to evaluate several of the aforementioned gravel correction and hydraulic property prediction methods, a laboratory experiment was designed to test various well-graded gravelly materials. An alluvial material sample was used to fabricate eight soils with various particle size distributions. The primary sample matrix for all testing was chosen to be the fine-earth fraction, less than 4.75 mm in diameter. Additional soil materials were then fabricated in which either part of the fine-earth fraction was removed, or gravel material ranging from 4.75 mm to 19 mm diameter was added. Test results show that saturated hydraulic conductivity (K_{sat}) decreased with up to 30% gravel content but increased by orders of magnitude at higher gravel contents. Moisture retention characteristic (MRC) data showed that the air entry value decreased with increasing gravel contents, although the amount of retained water only slightly decreased as gravel content increased. Depending on how the MRC data is interpreted, the predicted unsaturated hydraulic conductivities either showed increasing hydraulic conductivity as gravel contents increased, or the values converge.

The measured hydraulic property data could not be accurately predicted using published correction factors, or by other prediction methods that use particle size distribution data. Consequently, removing the gravel fraction could result in significant error in the prediction of mine waste drainage behavior and the performance of cover systems. It is recommended that published correction factors not be used unless the sample is similar in gradation and bulk density to the soils tested by the published method.

Additional Key Words: Moisture retention characteristic, gravel correction factor, ,

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Introduction

Effective closure of potentially acid generating mine wastes requires an understanding of how unsaturated flow occurs in the mine waste and through any proposed cover material system. For example, planning for mine waste closure could require predicting heap leach draindown, the performance of evapotranspirative cover systems, and long-term groundwater impacts from waste facilities. Due to the highly non-linear nature of unsaturated flow, models used to assist in closure design require robust hydraulic property data in order to generate reasonable predictions. Proper determination of mine waste and cover material hydraulic properties is therefore critically important in order to accurately model and predict flow behavior and reduce potential long-term liabilities.

Typical mine waste and cover materials in the western United States contain a large proportion of gravel (greater than 4.75 mm particle diameter) material. For this reason, measuring the moisture retention characteristics (MRC) and saturated hydraulic conductivity (K_{sat}) is difficult due to laboratory equipment size constraints and time requirements. A common commercial hydraulic laboratory practice is to remove all material greater than 4.75 mm in diameter from material samples and use small diameter cores to test the fine-earth fraction. The resulting measurements are then corrected for the soil gravel content with published correction factors such as Bouwer and Rice (1984). Bouwer and Rice determined that the K_{sat} of a sand and gravel mixture decreased with increasing gravel content and could be estimated from the K_{sat} of the sand fraction and void ratios of the mixture and sand fraction. However, it should be noted that the Bouwer and Rice study was based on testing bi-modally graded materials containing uniform large rock fragments (> 7.5 cm diameter) and homogeneous sand.

Other authors (i.e. Fredlund, 1999, Schaap et al., 2001, Mbonimpa et al., 2002, Aubertin et al., 2003) have proposed various methods to predict the K_{sat} and MRC from basic physical property data such as particle size distribution and bulk density. The accuracy of these methods are greatly improved if the hydraulic properties of similar material types are known (Fredlund et al., 2002), or if some moisture retention data is available (Schaap et al., 2001).

In order to evaluate several of the aforementioned gravel correction and hydraulic property prediction methods, a laboratory experiment was designed to test various well-graded gravelly materials. An alluvial material sample was used to fabricate eight soils with various particle size distributions. The primary sample matrix for all testing was chosen to be the fine-earth fraction, less than 4.75 mm in diameter. Additional soil materials were then fabricated in which either part of the fine-earth fraction was removed, or gravel material ranging from 4.75 mm to 19 mm diameter was added. Laboratory testing for each sample included K_{sat} and MRC determination and estimation of the unsaturated hydraulic properties using the methods of van Genuchten (1980) and Mualem (1976), and Fredlund and Xing (1994). Experimental results were then compared to hydraulic property predictions from only the fine-earth fraction using the correction factors published by Bouwer and Rice (1984). In addition K_{sat} and MRC values were also predicted from the particle size distribution data using the method of Mponimba et al (2002), and Fredlund (1999) and Fredlund et al (2002).

Materials and Methods

The materials tested were alluvial soil materials collected from the Nevada Test Site. The original soil sample was separated into different size fractions (Fig. 1) which were then used to fabricate eight soils with various particle size distributions. The primary sample matrix was assumed to be the fine-earth fraction, that is, material passing the #4 mesh, (all particles < 4.75 mm in diameter). Soil materials were then fabricated in which either part of the fine-earth fraction was removed, or gravel material was added.



Figure 1. Different size fractions of materials



Figure 2. 15-cm diameter by 30 cm cells

Samples were prepared by weighing the appropriate amount of each particle size fraction needed to prepare the desired particle size distribution (Fig. 3). Two different fine-earth samples were fabricated by removing the 4.75 mm to 2 mm diameter fraction and the 2 mm to 0.85 mm diameter fractions, respectively. The other six distributions contained the fine-earth fraction (< 4.75 mm diameter) with 0%, 30 %, 40%, 50%, 60 % and 80% gravel fractions, respectively. The gravel fractions added were not of uniform particle size, rather they represented well graded > 4.75 mm to < 19 mm diameter material, such as would be expected in most alluvial basin-fill environments.

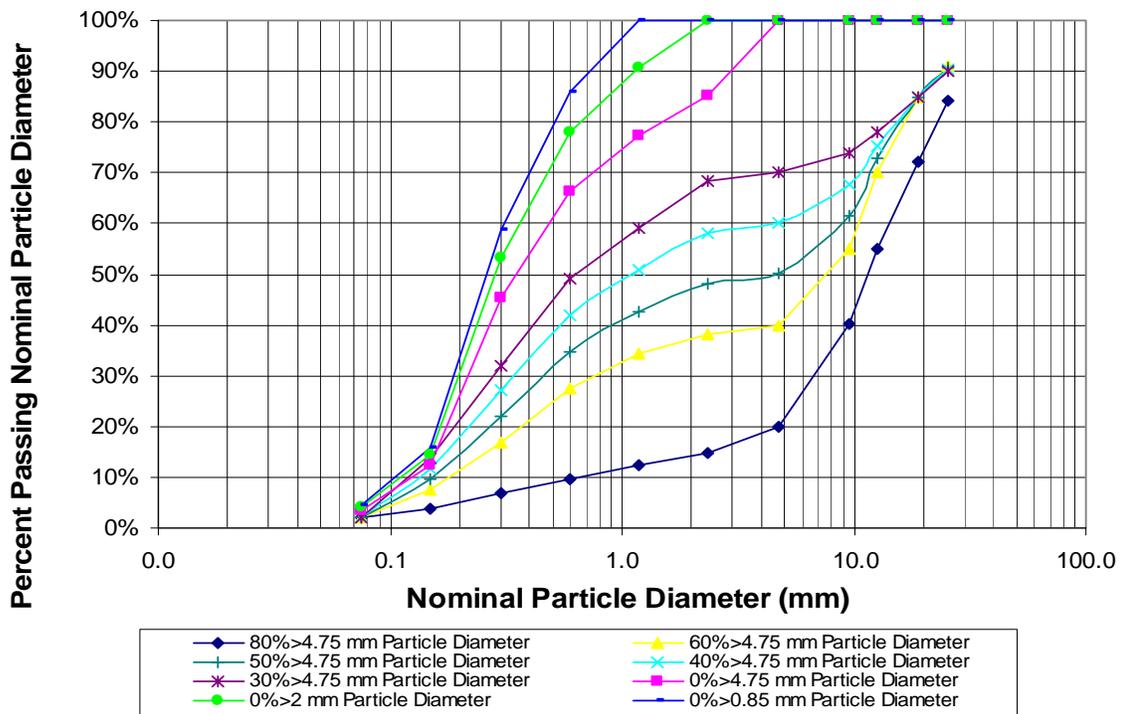


Figure 3. Particle size distribution of fabricated soils.

Each fabricated sample was manually homogenized and split into the number of test replicates. The fine-earth fraction distribution (<4.75 mm, <2 mm, <0.85 mm) tests were conducted using small 61.7 mm diameter by 150 mm length cells. Soils with 30%, 40%, 50%, 60% and 80% gravel fractions were tested using 150 mm diameter by 300 mm length cells (Fig. 2). Each sample was replicated three times except for the 40 %, 50% and 80 % gravel distributions which were not replicated. To avoid particle-wall interferences, gravel particles were limited to an upper size diameter of 19 mm. Fine-earth samples were packed in 20 mm lifts at a target dry bulk density of 1.5 g/cm³, soils

with gravel were packed in 50 mm lifts at target densities of 1.6 g/cm³. The 30% gravel sample was packed to a slightly higher bulk density due to settling during the saturated hydraulic conductivity (K_{sat}) testing.

Cells were saturated using upward infiltration, K_{sat} measurements were conducted using constant head methods (Method 3.4.2.2, MOSA, 2002). After measuring the K_{sat} , the moisture retention characteristic (MRC) curve was determined using six pressure steps. Hanging column methods were used up to -100 cm of tension followed by Tempe Cell pressure plate extraction up to -1000 cm of tension (Method 3.2.2, MOSA, 2002). Three additional gravimetric water content-matric potential points in the range of -10,000 to -120,000 cm (-980 to -11,800 Kpa) were obtained using a WP-4 water activity meter (Decagon, WA) on the <4.75 mm fraction. Estimated volumetric water contents at the dry points for the gravel samples were extrapolated from the WP-4 data by assuming that the water content decreases in proportion to the gravel fraction, that is, we assumed the gravel contains no water.

The unsaturated hydraulic flow properties were estimated from the measured data using two different approaches. The first approach used the van Genuchten (1980) and Mualem (1976) methods to predict the moisture retention characteristic curve and relative unsaturated hydraulic conductivity function (Mualem-van Genuchten). The computer code RETC4 (van Genuchten, et al., 1997) was used to calculate α and N , the van Genuchten parameters needed to estimate the hydraulic conductivity function (K_{unsat} vs moisture content or tension), from the negative pressure potential (tension) and water content relationship. The residual water content (the water content at which flow ceases) was adjusted in cases where calculated values were unreasonably low. Finally, the RETC parameter M was specified to equal $1-1/N$. The second approach to modeling the measured data used the computer program SOILVISION 3.0 (SoilVision Systems Ltd. 2001) to estimate the moisture retention characteristic function via the method of Fredlund and Xing (1994) and the unsaturated hydraulic conductivity function via the method of Fredlund et al. (2002).

Finally, the particle size distribution and bulk density data were used to predict the K_{sat} via the method of Mponimba et al. (2002), and the moisture retention characteristic curve using the method of Fredlund (1999) as incorporated into SOILVISION 3.0 (2001).

Results

Saturated Hydraulic Conductivity

Measured K_{sat} results are shown in Fig. 4 and compared to the predicted K_{sat} using the methods of Bouwer and Rice (1984) and Mponimba et al. (2002). K_{sat} values for the three fine-earth distributions were similar and varied between 6.7×10^{-4} and 9.4×10^{-4} cm/sec. The average K_{sat} decreased to 3.0×10^{-4} cm/sec with 30% gravel content. This K_{sat} decrease is consistent with the “bricks and mortar” model of Bouwer and Rice (1984) whereby gravel acts as a barrier to flow. However, using the void ratio correction factor for percent gravel provided by Bouwer and Rice (1984), a K_{sat} of only 6.2×10^{-4} cm/sec is predicted (Fig. 4) suggesting that increased bulk density also contributed to the decrease in K_{sat} .

Conversely, when the gravel content increased above 30%, the measured K_{sat} values increased significantly (Fig. 4). The dramatic increase in K_{sat} indicates that gravel contents above 30% resulted in highly conductive interconnected pores. Using the Mponimba (2002) method to predict K_{sat} from particle size distributions and bulk density values consistently over-predicts K_{sat} , although similar trends are observed (Fig. 4).

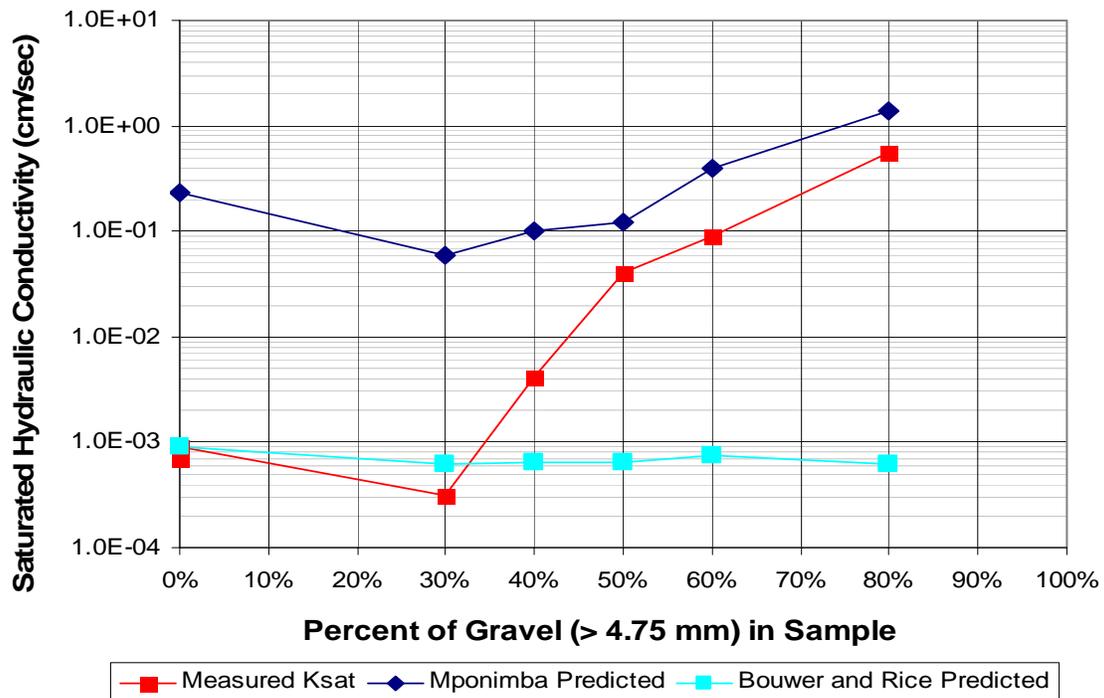


Figure 4. Measured saturated hydraulic conductivity vs. percent of gravel in sample.

Moisture Retention Characteristic and Predicted Unsaturated Hydraulic Conductivity

MRC data and the predicted hydraulic conductivity vs matric potential function using Mualem-van Genuchten method for the <0.85 mm, <2 mm and <4.75 mm diameter materials are shown in Fig. 5. The MRC and hydraulic conductivity functions for the three fine-earth fraction distributions are similar suggesting that the pore size distribution is similar for these materials.

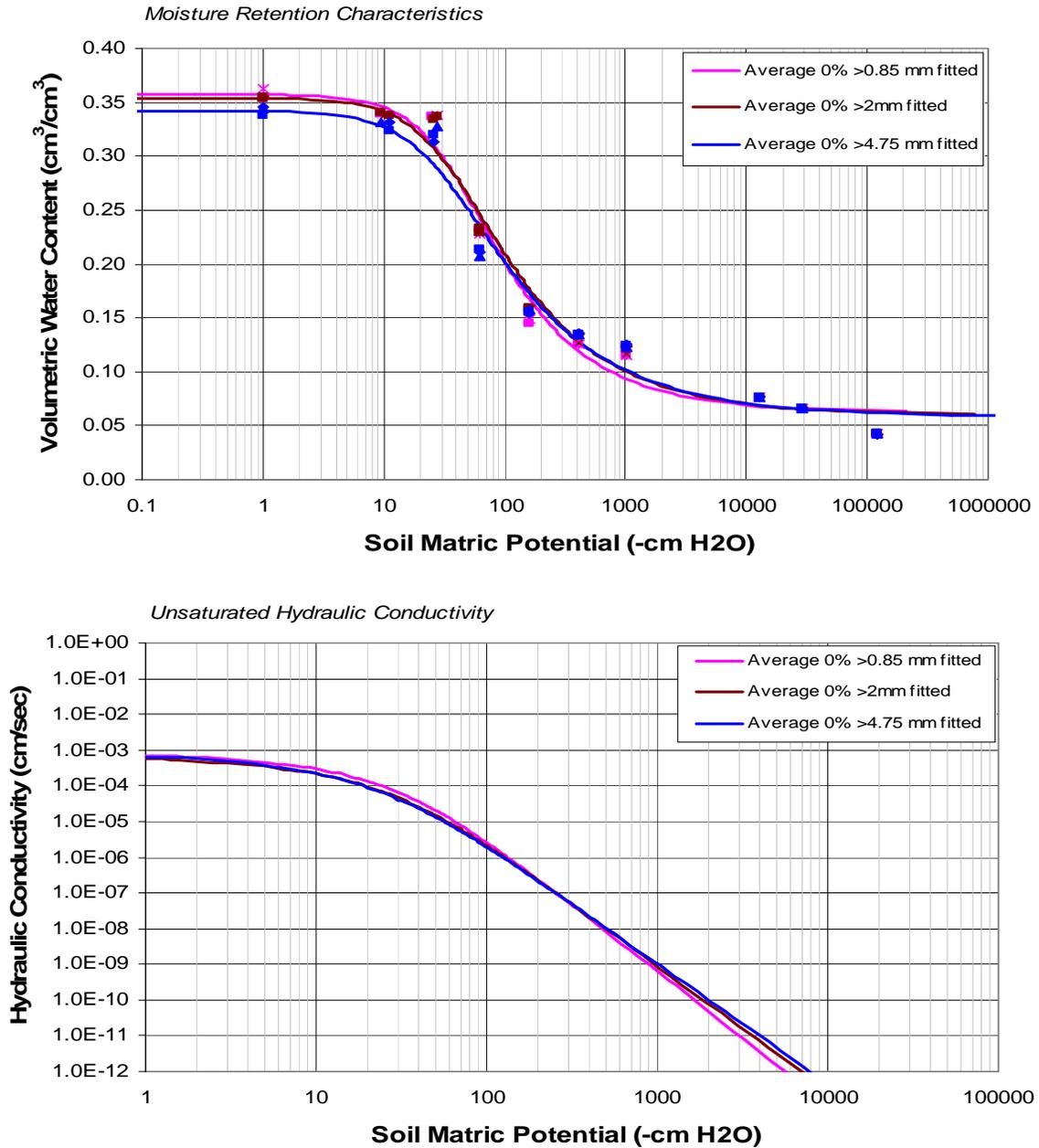


Figure 5. Mualem-van Genuchten predicted MRC curves and hydraulic conductivity-matric potential functions for fine-earth samples.

Figure 6 shows the MRC data and predicted hydraulic conductivity vs matric potential functions using Mualem-van Genuchten. The MRC data indicates that increasing gravel contents above 30% causes the air entry value to decrease as is consistent with larger pore sizes that drain under less negative matric potentials. However, the relative decrease in moisture retention with increasing gravel content was less than expected, most likely due to the well graded material maintaining small pores.

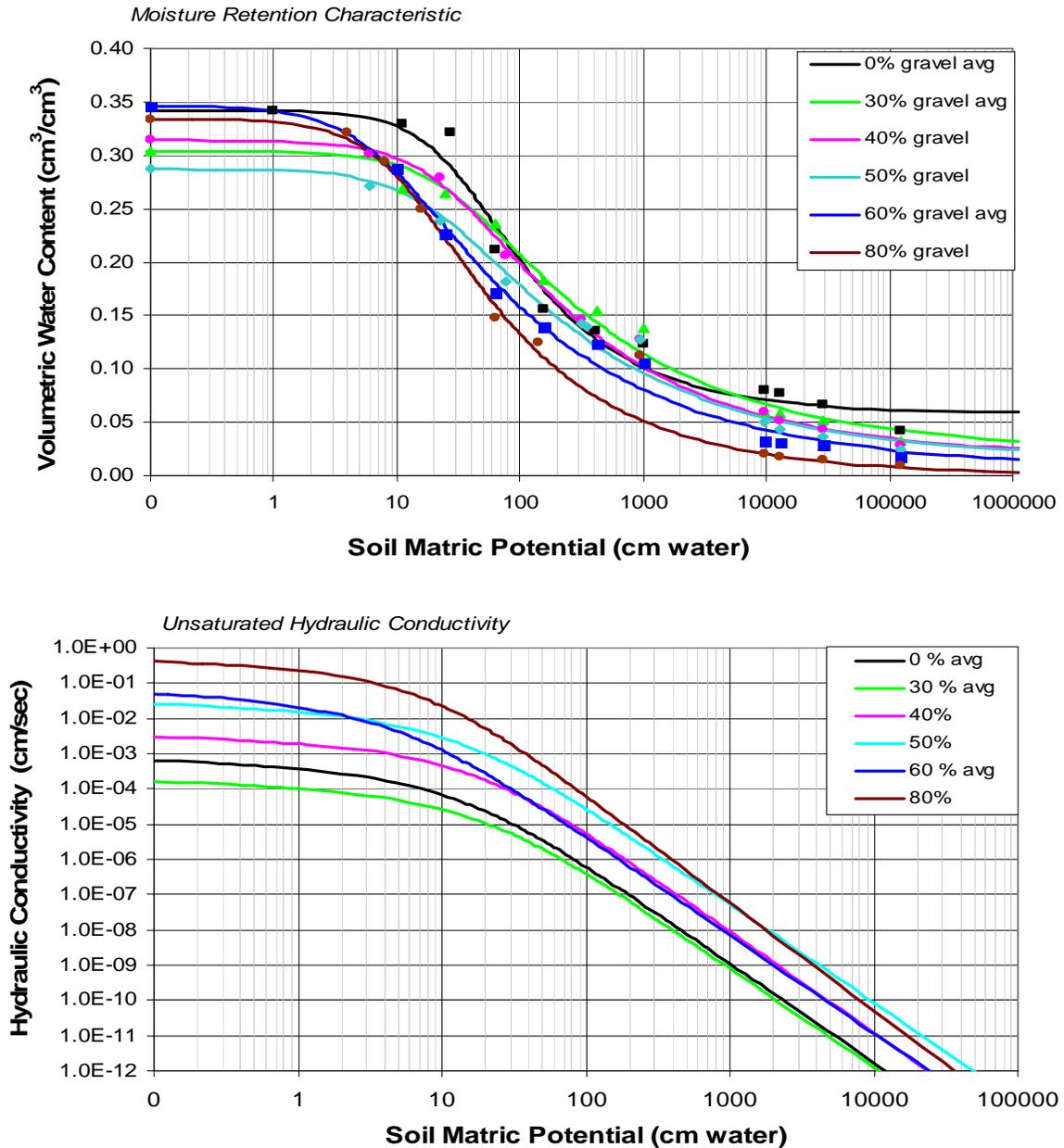


Figure 6. Mualem-Van Genuchten predicted MRC curves and hydraulic conductivity-matric potential functions for all samples.

It should be noted that more moisture is retained at the -330 cm and -1000 cm (10 and 32 kPa) pressure points than would be expected by the drying trend shown with WP-4 dry point data. This is an artifact of the Tempe cell procedure and results from the very low hydraulic conductivity of the materials at those pressure steps. For example, the -1000 cm pressure point was ceased when daily outflow rates decreased below a 0.05% change in volumetric moisture content. Projecting the WP-4 dry point and the outflow flux rates indicates that the -1000 cm data point could have required another three to six months for full drainage to occur. Consequently, the -1000 cm pressure step cannot be considered to be at equilibrium. Nevertheless, the relative convergence of water content between 10 and 14% at the -1000 cm pressure step, and the consequent divergence from the predicted values, suggests that the moisture retention for the high gravel content soils is only slightly less than the low gravel content soils at low flux rates (i.e. 1×10^{-7} cm/sec).

The modeled (Mualem) hydraulic conductivity functions of the different samples (Fig. 6) remain parallel throughout the matric potential range, that is, the difference in hydraulic conductivity remains fairly constant between the different soils and the high gravel content soils maintain higher conductivities than the low gravel content soils even in the dry range. The parallel increase in hydraulic conductivity with increasing gravel content is in direct contrast to the Bouwer and Rice (1984) prediction of parallel decrease in hydraulic conductivity. Moreover, the predicted unsaturated hydraulic conductivities for the very gravelly soils (> 40%) are contrary to capillary tube theory which would predict very low hydraulic conductivities at more negative matric potentials due to the lower quantity of small diameter pores. However, the relative convergence of the moisture content data for the 60% and 80% gravel content soils at the higher pressure steps (> -300 cm) suggests that the unsaturated hydraulic conductivity of these soils actually decreases rapidly and converges with the other soils.

Figure 7 shows the modeled MRC data and the predicted unsaturated hydraulic conductivity functions using the method of Fredlund and Xing. The 0%, 40% and 80% gravel content soils show similar predicted MRC curve and hydraulic conductivity functions as the Mualem-van Genuchten approach. However, the 30%, 50% and 60% gravel content soils show lower predicted air entry values and decreasing hydraulic

conductivity due to SoilVision (2001) initializing the saturated water content as the total porosity rather than the actual measured saturated water content.

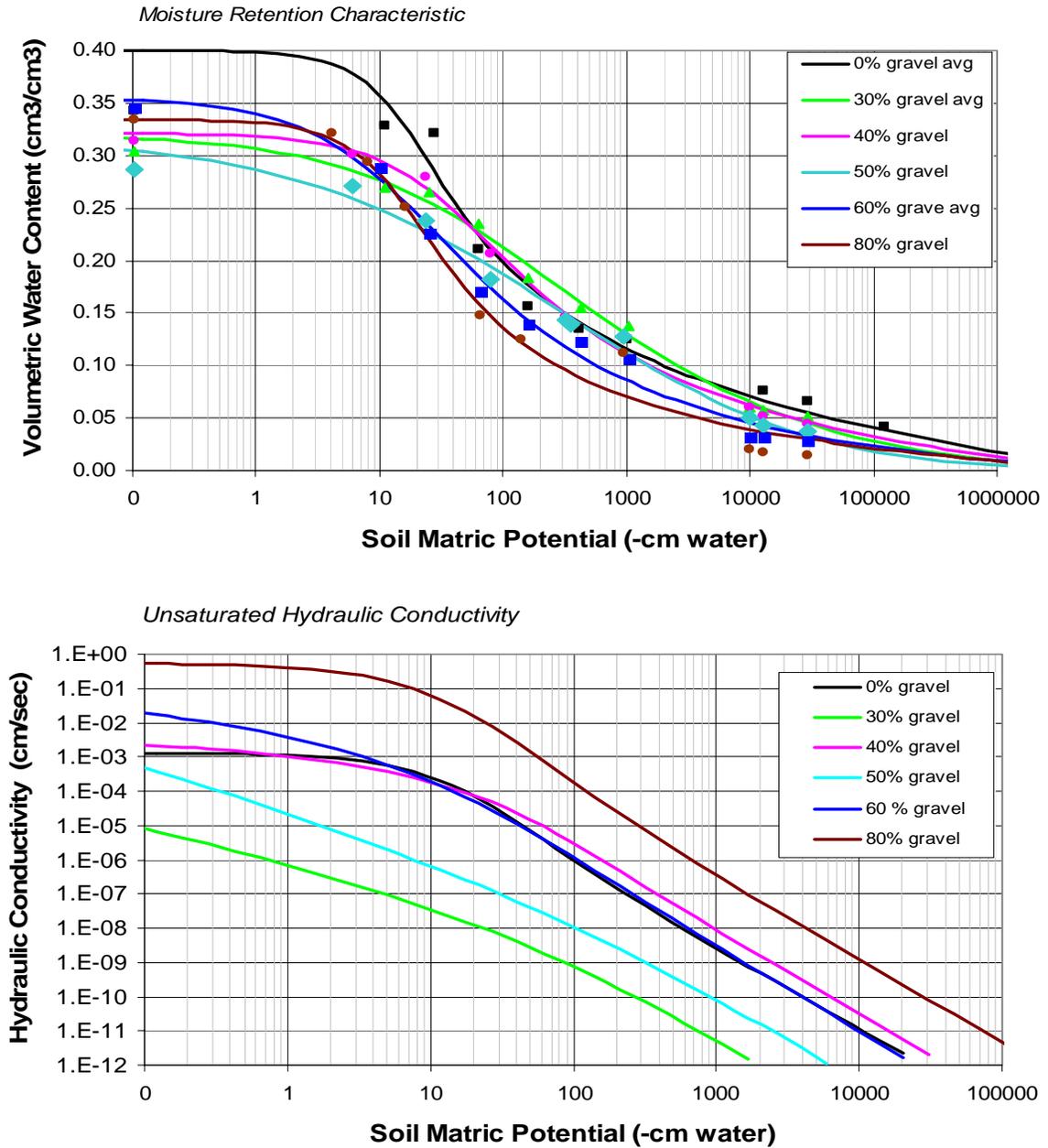


Figure 7. Fredlund and Xing predicted MRC curves and hydraulic conductivity-matric potential functions for all samples.

Finally, Fig. 8 depicts measured MRC data and predicted MRC curves using the particle size distribution prediction method of Fredlund (1999) via the neural net estimation approach contained in SoilVision (2001). Poor agreement is shown for almost

all of the MRC data as the predicted MRC curves show lower air entry values and more rapid drainage. This is most likely due to both the high gravel content and well-graded nature of the material. Fredlund et al. (2002) noted that it is particularly difficult to predict the MRC for soils that contain large amounts of coarse particles and few fines (< #200 mesh). Swanson et al. (1999) also found that Fredlund (1999) particle size distribution based MRC predictions performed poorly with well graded materials and required additional calibration using MRC data from very similar particle size distribution soils.

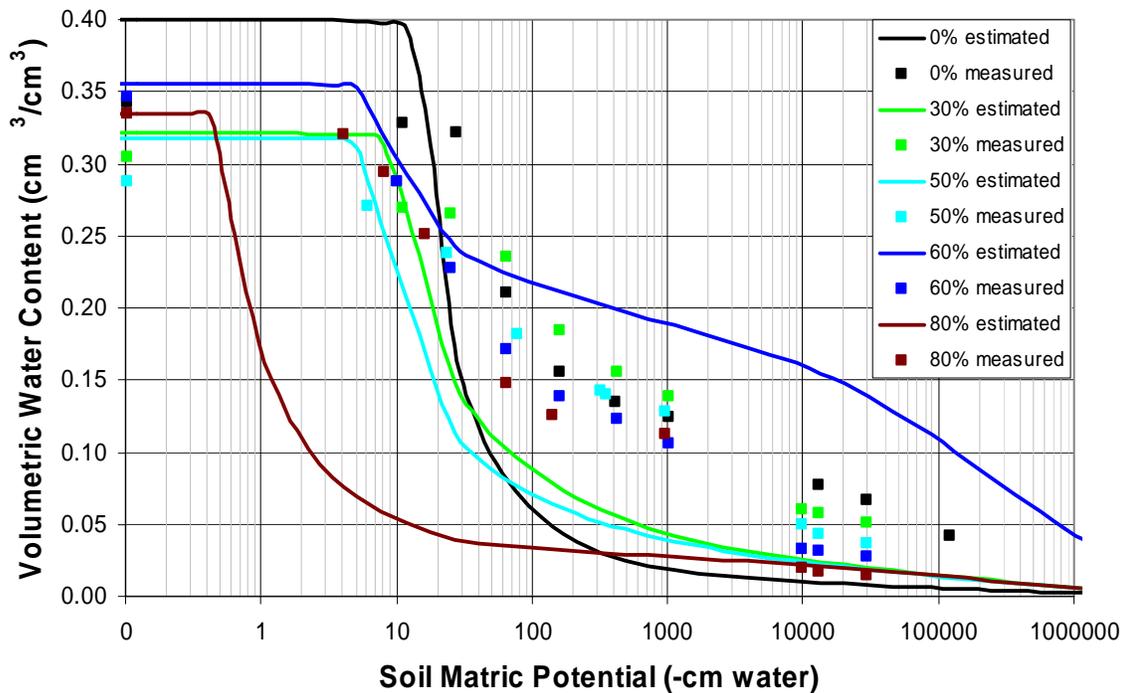


Figure 8. Fredlund (1999) predicted MRC curves based on the sample particle size distribution.

Discussion

In contrast to previous findings by Bouwer and Rice (1984), the presence of gravel contents greater than 30% resulted in significant increases in K_{sat} . The data indicate that for these materials, interconnected macropores created by > 4.75 mm diameter particles can increase the K_{sat} by several orders of magnitude. The particle size distribution approach proposed by Mponimba (2002) significantly overpredicted the K_{sat} for the 0% to 40% gravel content soils. The lower measured K_{sat} values are most likely due to

smaller pore sizes created by the well graded soil materials that cannot be accounted for in the Mponimba model.

The well graded particle size distribution also appeared to result in greater moisture retention at high negative pressure steps. Although the final -1000 cm Tempe cell pressure step was not at equilibrium, the low outflow rates indicate very low unsaturated hydraulic conductivities at the -1000 cm pressure step. The predicted Mualem-van Genuchten and Fredlund and Xing MRC curves showed relatively good agreement with the data of this study, however, the predicted hydraulic conductivity functions seem to show unreasonably high unsaturated hydraulic conductivities for the very gravelly materials at more negative matric potentials.

Several factors may cause the apparent poor prediction of the relative conductivity methods. For simplicity, the discussion will be limited to the Mualem-van Genuchten model. Schaap and Leij (2000) have noted that order of magnitude error in hydraulic conductivity could be associated simply by optimization errors in the van Genuchten parameters, in particular at low negative matric potentials (i.e. > -10 cm). Moreover, the high K_{sat} values for soils with above 40% gravel content are clearly due to macropore flow. Since the Mualem method correlates the hydraulic conductivity function to K_{sat} , the exceedingly high K_{sat} values could result in exceedingly high unsaturated hydraulic conductivity values. Application of a dual porosity MRC model (i.e. Durner, 1994; Zhang and Chen, 2005) is insufficient to significantly change the hydraulic conductivity function since it also uses the Mualem relative conductivity approach. Therefore, it is most likely that the Mualem assumptions of pore continuity and connectivity may not be satisfied as very large pores that contribute orders of magnitude increases in K_{sat} may only constitute a small fraction of the total porosity and may not be connected at water contents below saturation. Very recently, Schaap and van Genuchten (2005) proposed an empirically based correction to the Mualem-van Genuchten model to account for the influence of macropores in the near saturation range. Testing of this method on the data set was not feasible by the time of publication.

Even if near-saturated hydraulic conductance data points could be collected to reduce the influence of macropore flow, the continuity principle may also not be valid at very negative pressure potentials in the experimental soils. For example, if pores became

isolated under unsaturated conditions, flow could effectively cease even at relative high water contents (i.e. $0.10 \text{ cm}^3/\text{cm}^3$). In other words, water may still be held on gravel surfaces or within matrix pores, even though not available for flow. This effect can be approximated by ignoring the WP-4 data and increasing the residual water content values to between 6 and 10% for the van Genuchten parameter estimation. Results for this approach are shown in Fig. 9.

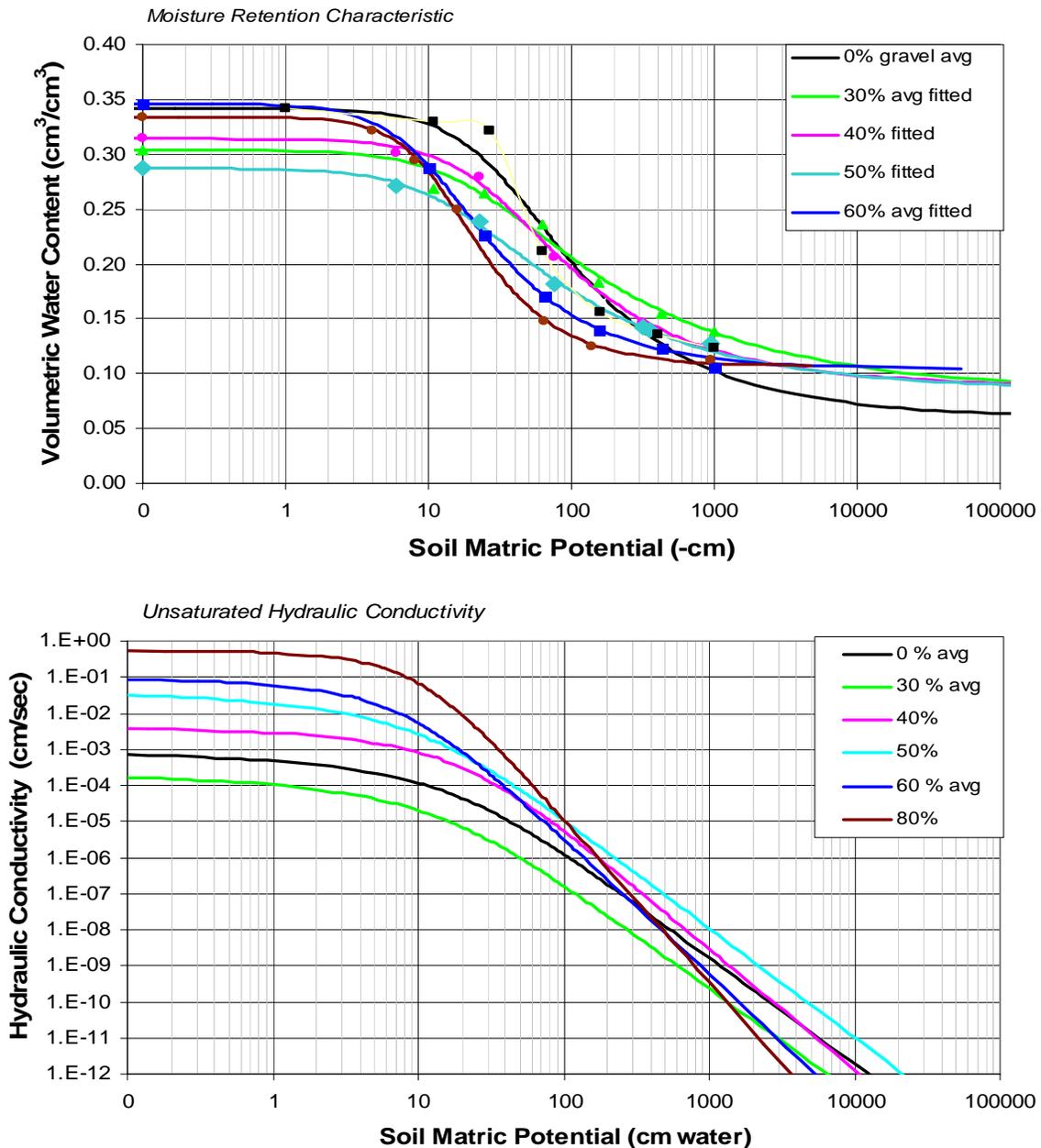


Figure 9. Results of fixing the residual water content values between 6 to 10% for the Mualem-Van Genuchten predicted MRC curves and hydraulic conductivity-matric potential functions.

These Mualem predicted hydraulic conductivity functions using high residual water contents for the gravel soils showed similar hydraulic conductivity (Fig. 9) with the previous predictions (Fig. 6) for the 0% and 30% gravel content soils. Whereas, the fitted MRC curves for the more gravelly soils, show a marked decrease in hydraulic conductivity at matric potentials below -300 cm. Although there is currently no theoretical basis for this approach, these predicted values are more consistent with general concepts of unsaturated flow, and suggest that residual water contents in well graded gravelly soils could be high.

Conclusions

The results of this study did not replicate the findings of Bouwer and Rice (1984), and the hydraulic properties of the different fabricated soils also could not be adequately predicted using the methods of Mponimba (2002) and Fredlund (1999). The results indicate that the laboratory practice of only testing the fine-earth fraction and then using correction factors, or simply using predictive models based on particle size distribution could result in orders of magnitude error at both the near-saturated and very dry moisture conditions. Moreover, the predicted hydraulic conductivity functions derived from applying the Mualem (1976) and Fredlund et al. (1994) methods to the measured K_{sat} and MRC data were not consistent and showed exceedingly high predicted unsaturated hydraulic conductivities for the very gravelly material.

The hydraulic properties of the fine-earth material samples were most likely controlled by the smaller diameter materials (<0.85mm). K_{sat} decreased with up to 30% gravel content (>4.75 mm), but increased dramatically at higher gravel contents. This is in direct contrast to the findings of Bouwer and Rice, 1984, who found that K_{sat} was inversely proportional to gravel contents up to 70% when uniform gravel size and fine-earth fractions were used. In this study, as well graded gravel contents increased, larger diameter pores were created resulting in increased K_{sat} .

Moisture retention characteristics were also affected by increasing gravel contents. MRC data showed that the air entry value and the amount of retained water at low matric potentials decreased with increasing gravel contents. Retained water in all of the different gravel soils appear to converge at tensions less than -1000 cm due to very low

hydraulic conductivities at these pressures. In addition, the predicted unsaturated hydraulic conductivity using standard pore size distribution models showed greater unsaturated hydraulic conductivities with increasing gravel contents at more negative matric potentials. These unusual predictions suggest that assumptions of hydraulic and pore continuity may not be valid in gravelly soils at the wet (near-saturated) due to macroporosity and under moderately dry conditions due to isolated pores. It also suggests that the (residual) moisture content, at which flow ceases, could be relatively high in gravelly soils.

Based on these results, there are several potential consequences to unsaturated flow modeling predictions if gravel content is removed during hydraulic property testing:

- a) The amount of flux through a cover system could be underestimated due to the prediction of lower unsaturated hydraulic conductivities under wet conditions than may actually occur at low negative pressure potentials (i.e. > -50 cm).
- b) The long-term drainage rates (and subsequently the amount of drainage) from a heap leach facility could be over-estimated due to the prediction of more negative air entry values than may actually occur, and subsequent higher unsaturated hydraulic conductivities at moderate matric potentials (i.e. -50 to -300 cm).
- c) The amount of moisture storage in a waste rock facility available before flow occurs (the field capacity) could be underestimated due to the prediction of lower residual moisture contents than may actually occur. Moreover, changes in flux that could occur with a relatively minor increase or decrease in moisture content could also be underestimated.

In summation, the removal of gravel material from soil samples for hydraulic property testing is not be recommended unless the sample is similar in gradation and bulk density to the soils used to develop the published correction factor.

These data raise a number of questions that deserve further investigation. The following alternative procedures are recommended:

- Direct unsaturated flow measurements in large sample cores for gravelly material at near-saturated and very dry conditions.

- Direct unsaturated flow measurements in small sample cores for fine-earth materials at near-saturated and very dry conditions.
- Large sample core tests on 30%, 40%, 60% and 80% mixtures at high bulk densities to assess the affect of increasing bulk density on hydraulic properties.
- Large sample core tests on 10% and 20% gravel soil mixtures to determine the correlation between gravel content and hydraulic properties between 0% and 30% gravel.

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