

# **Calibration and Predictive Performance of a Two Domain Empirical Metal Recovery Model**

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## **Abstract**

An empirical metal recovery model that accounts for metal dissolution and transport in two domains is presented. The domains are defined as regions of the leach pad with different reaction kinetics and transport properties: a “fast” domain and a “slow” domain. The fast domain is the fraction of the ore that readily releases gold to the leaching solution, and the slow domain releases gold more slowly. The model uses algorithms to allow the estimation of model parameters from column test recovery, particle size distribution and head grade data. To test the functionality of the metal recovery model, it was calibrated to available physical, metallurgical and column testing data from 18 different ore samples, and column test conditions (irrigation rate, reagent concentrations) were evaluated for relationships to model parameters and results. Estimated model parameters and related functions were successfully applied to predict column recovery data for twenty separate ore samples with minimal adjustment. The model was also able to reasonably predict gold recovery from an operational heap after adjusting model parameters to account for reduced recovery at the heap scale, and one parameter (penetration depth) may account for differences in leaching processes that inhibit dissolution or transport. The model results indicate that the recovery model can be used as a relatively simple and reliable operational tool to provide information for management of ore inventories and solution requirements in active heap leach facilities. This model has been filed for patent protection.

## Introduction

Heap leach system dynamics are complex, involving multiphase interactions between solid, liquid and gas phases (Bartlett, 1998; Petersen and Dixon, 2007). There are also multiple chemical reactions occurring in the heap leach system. Furthermore, ores can contain numerous mineral types and are comprised of particles that can vary in scale from microns up to meters (Bartlett, 1998; Bouffard and West-Sells, 2009).

Models applied to describe the heap leach system can be used as operational tools to track and predict metal recovery in heap leach operations. Model input requirements vary with model complexity. Consequently, the selection of an appropriate model should be dictated by the level of available data and a desired level of model accuracy. Readily available measurements of heap structure and ore physical, metallurgical, and hydrological properties obtained from laboratory data may be sufficient inputs. Typical inputs include ore type, head grade, particle size distribution, input leach kinetics, wetting rate, heap lift configuration, irrigation rate, and time (e.g. Luiz et al., 2003; Mellado et al., 2009; Bouffard and Dixon, 2008). Gold recovery and pregnant solution gold concentration are the fundamental model outputs. This paper describes the development of a metal recovery model for use as an operational tool at an open-pit, run-of-mine (ROM), heap leach gold mine.

## Model Description

The metal recovery model estimates cumulative gold recovery using first order rate terms to describe gold dissolution and transport in two domains. One domain represents the fraction of gold that is readily released and transported to the collection system. Gold recovered from this domain represents the majority of initial gold recovery. For simplicity, this gold fraction is referred to as the fast leaching domain. The second domain, the slow leaching domain, represents the fraction of the gold that takes longer to be dissolved and transported due to kinetic limitations such as gold dissolution rates being diffusion limited or dissolved gold trapped in pore space that has low solution conductivity. Gold recovery rates from the slow leaching domain are low but represent the majority of late time recovery. Model equations and algorithms have been filed for US patent protection.

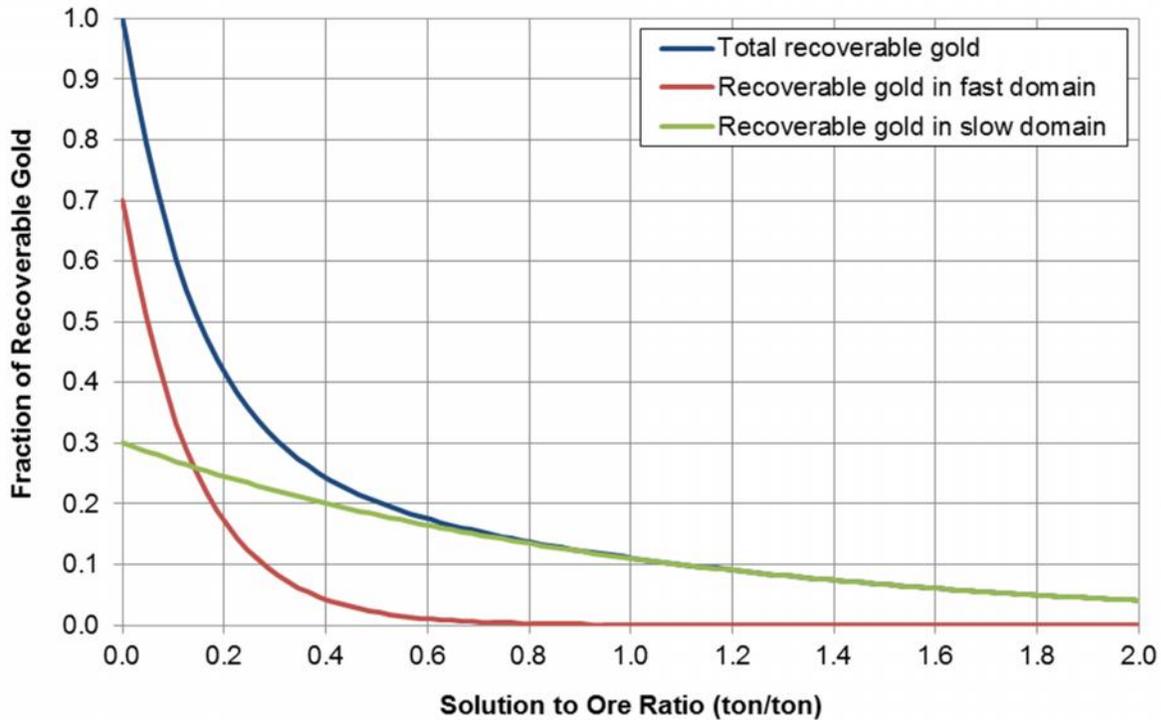
## Model Equation

The equation for recovered gold fraction as a function of the solution-to-ore ratio,  $R_{sr}$ , is:

$$R_S = R_m \times L \left[ 1 - F e^{-k_1 \frac{s}{r \times L}} - (1 - F) e^{-k_2 \frac{s}{r \times L}} \right] \quad (1)$$

where  $R_{max}$  is the maximum recoverable gold fraction as determined by laboratory assay,  $LE$  is the leaching efficiency,  $F$  is the fast leaching gold fraction;  $k_1$  is the first order rate coefficient that describes the dissolution and transport in the fast leaching domain;  $k_2$  is the first order rate coefficient that describes the dissolution and transport in the slow leaching domain;  $s$  is the mass of applied leaching solution and  $r$  is the mass of ore being leached. The ratio of  $s$  to  $r$  ( $s:r$ ) is directly related to time and leaching rate. The  $LE$  parameter accounts for variable field scale leaching characteristics.  $LE$  may be less than one, for example under conditions of solution flow bypassing ore; or may be greater than one, indicating delayed recovery of gold from underlying lifts.

Figure 1 presents an example of the recoverable gold remaining in each domain and the total recoverable gold as leaching occurs. The  $Fe^{-k_1\frac{s}{r}}$  term in Equation 1 represents the amount of gold remaining in the fast leaching domain. The term equals  $F$  at the beginning of leaching and decreases as  $s:r$  increases. The  $(1 - F)e^{-k_2\frac{s}{r}}$  term represents the amount of gold remaining in the slow domain. This term equals  $(1 - F)$  at the beginning of leaching and decreases as  $s:r$  increases. Both terms are subtracted from 1 to yield the fraction of recovered gold and then the amount is scaled by  $R_m$  to account for the maximum fraction of gold in the ore that can be recovered.

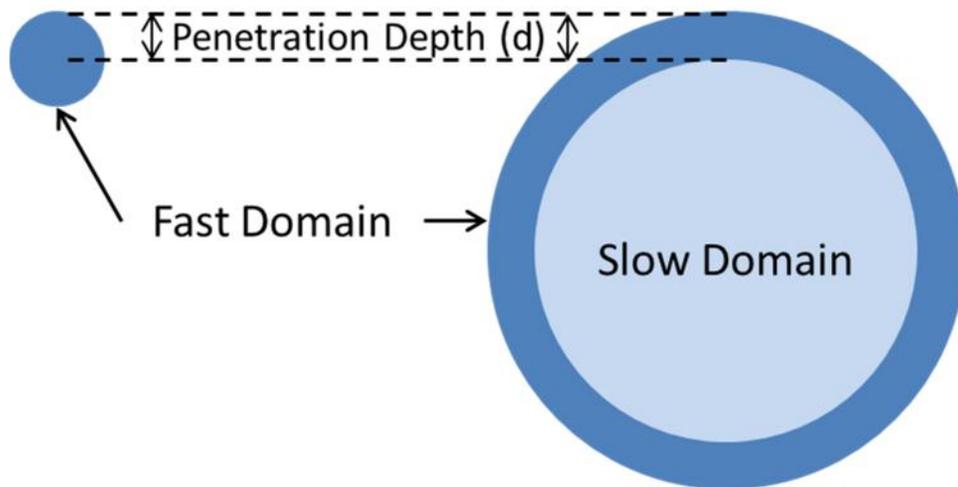


**Figure 1. Fraction of recoverable gold remaining in the slow and fast domains and the total recoverable gold remaining**

The gold in the fast leaching domain,  $F$ , is released quickly as shown by the steep slope of the fast leaching domain curve at  $s:r$  values near zero. The large change in the total fraction of gold remaining in the ore coincides with the fast leaching domain release, demonstrating that the fast leaching region is the predominant source for early gold recovery. In Figure 1,  $F$  equals 0.7 and gold in the fast leaching domain is mostly recovered by  $s:r$  of 0.5 and at this reflection point the release of gold in the slow leaching domain becomes the primary source of released gold.

### Early Release Fraction (ERF) Algorithm

Equation 1 is empirical and does not directly describe the physical processes of leaching, so it is desirable to derive a unique set of model parameter values that are based on ore physical properties. An algorithm that predicts model parameters based on the fraction of gold available for early release (early release fraction (ERF)) is used, but not detailed herein to maintain proprietary status. The ERF algorithm is based on the principle that fast domain gold is mostly in small ore particles and also from the surface of larger ore fragments. The ERF algorithm estimates the volume of early release gold and is functionally equivalent to  $F$ . Figure 2 shows how different size particles contribute to the ERF. After leaching begins the lixiviant will penetrate a small distance into the ore particle and dissolve gold. While similar in concept to the shrinking core model (Roman et al., 1974) the ERF algorithm does not calculate reactant diffusion and metal dissolution. For particles with a small diameter (e.g. particle on left in Figure 2) gold is readily recovered from all locations within the particle and thus the entire particle contributes to the ERF. The ERF contribution from particles with a diameter greater than the penetration depth ( $d$ ) is restricted to the outer region of the larger particle (e.g. particle on right in Figure 2).



**Figure 2. Schematic of penetration depth and Early Release Fraction (ERF)**

The fast domain reaction rate,  $k_1$ , is influenced by several properties that may inhibit gold dissolution and transport including rock type, ore hydraulic properties, mineralogy, the amount of ERF that is in smallest size fractions and reagent application rate to name a few. Nonetheless, the fraction of ERF associated with particle size classes <0.5 inches is shown to be correlated to  $k_1$ , and the ERF for all size fractions is correlated to  $k_2$ .

## Results

Column testing data and ore physical property data were used for metal recovery model development. Column test data sets were limited to ROM ore samples or ore samples with crush size < 4 inches to be more representative of the ROM material placed on the leach pads. A total of 18 ore samples were used for model calibration and development of functions relating ERF to the rate parameters  $k_1$  and  $k_2$ .

### Calibration and Parameter Correlation

The model was calibrated to individual column test data sets and model parameters were developed by iterative steps:

1. Calculate ERF for each ore sample and assign model parameter  $F$  equal to ERF.
2. Calibrate the model to the column leaching data using the assigned  $F$  parameter and adjusting parameters  $k_1$  and  $k_2$ .
3. Develop correlation of calibrated  $k_1$  to ERF using results from step 2
4. Recalibrate the model to the column leaching data using the assigned  $F$  parameter and  $k_1$  equal to the value estimated from step 3 and then adjusting parameter  $k_2$ .
5. Develop correlation of calibrated  $k_2$  to ERF using results from step 4

The calculation of ERF determined a  $d$  value which provided the best overall correlation between model parameter values and ERF. Figure 3 presents the function relating  $k_1$  to ERF < 0.5”:

$$k_1 = 1.3936 \times e^{3.3 \times E_{<0.5''}} \quad (2)$$

Similarly,  $k_2$  was found to be related to the ERF. Figure 4 presents the function relating  $k_2$  to ERF.

$$k_2 = 0.0410 \times e^{5.2 \times E} \quad (3)$$

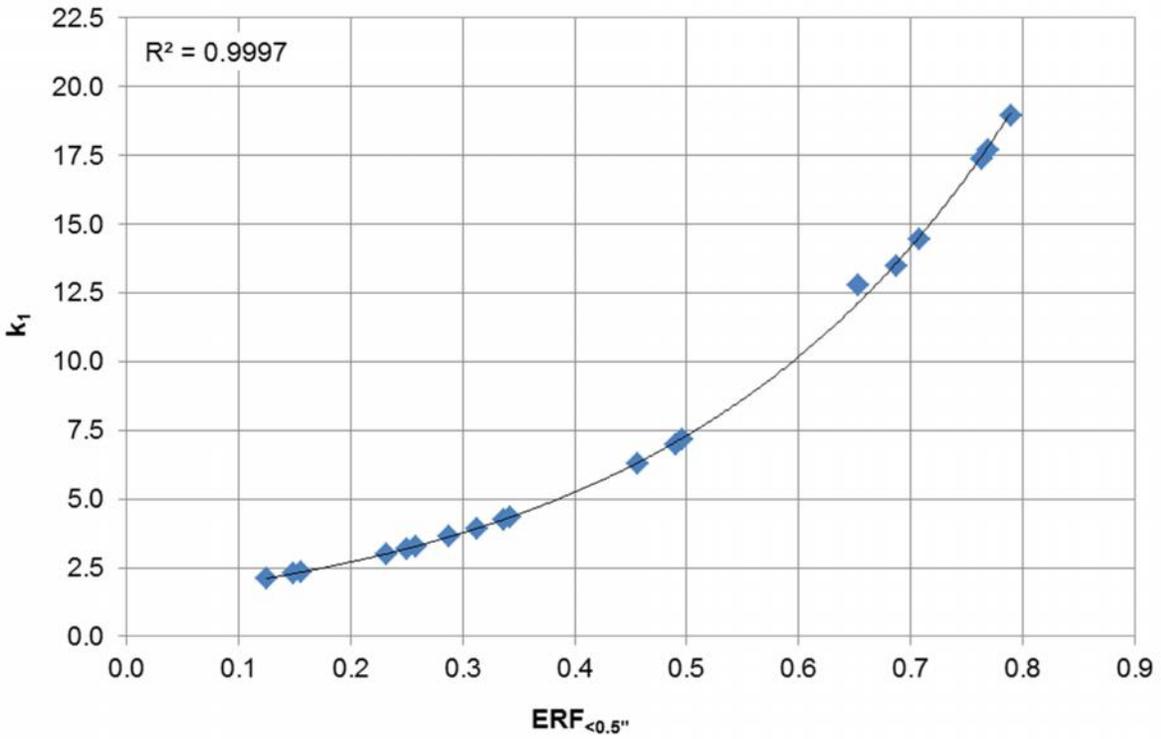


Figure 3.  $k_1$  as a function of  $ERF_{<0.5''}$  (data from 18 calibrated models)

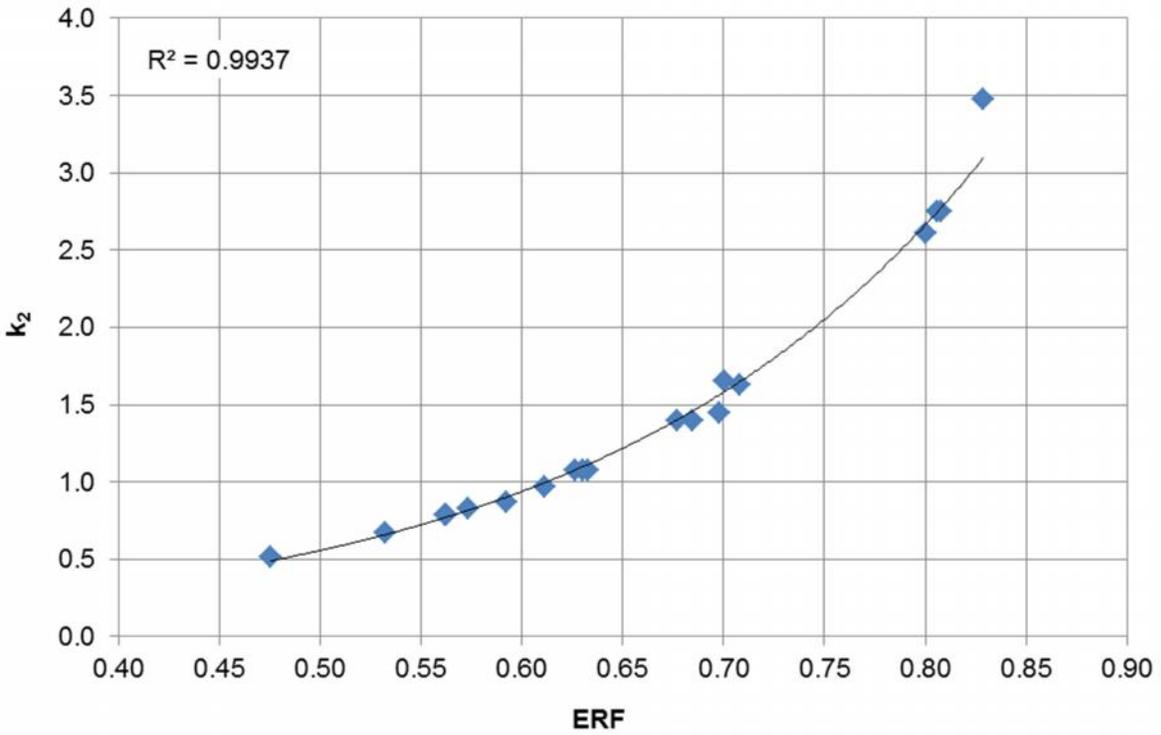


Figure 4.  $k_2$  as a function of  $ERF$  (data from 18 calibrated models)

## Model Performance

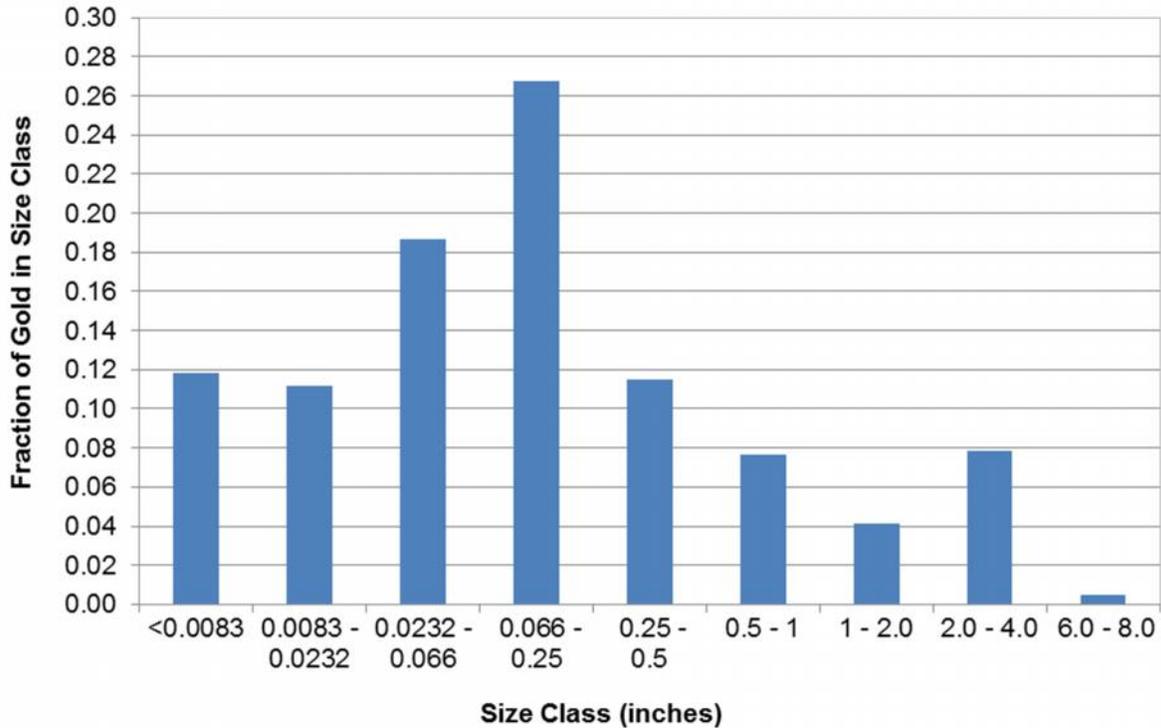
The degree of confidence in model estimates of gold recovery using parameter values determined from the ERF algorithm and Equations 2 and 3 was evaluated for two independent column leaching data sets, Ore Type 1 and Ore Type 2.

### *Ore Type 1*

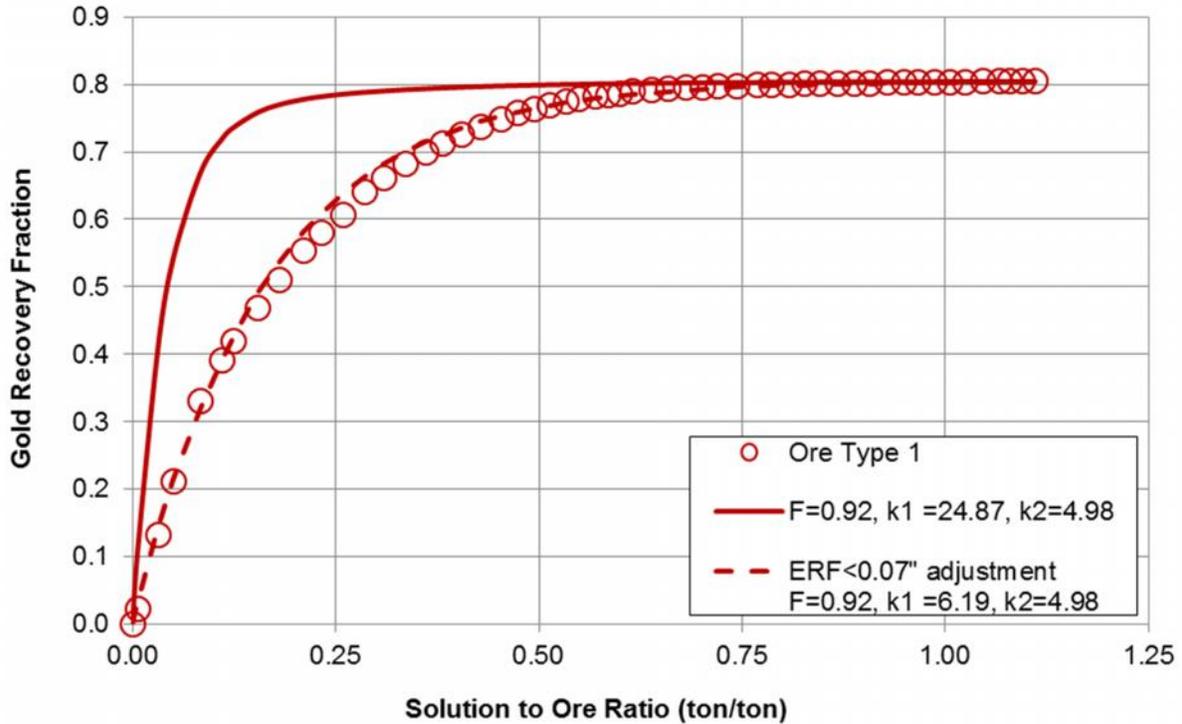
Figure 5 presents the fraction of recoverable gold in each size class for the Ore Type 1 sample. Eighty percent of the Ore Type 1 sample gold mass is associated with ore sizes < 0.5 inches.

Figure 6 presents model predicted and measured gold recovery for the Ore Type 1 sample. The predicted recovery was greater than measured recovery (RMSE = 0.0892) because the ERF<0.5” was very high (0.87). The ERF<0.5” may overestimate  $k_f$  because Equation 2 was derived from samples with a smaller fraction of gold associated with the <0.5 inch size classes than is observed for the Ore Type 1 sample.

To account for the larger fraction of gold in the smallest size classes, the  $k_f$  model parameter value was derived by adjusting ERF<0.07” in Equation 2. This modification to  $k_f$  resulted in an improved prediction of gold recovery (RMSE = 0.0077) and indicates a potential limitation with the ERF<0.5” relationship if a large fraction of gold is associated with ore particles much smaller than 0.5 inches.



**Figure 5. Gold distribution by particle size class for Ore Type 1 sample**



**Figure 6. Model predicted and measured recovery for Ore Type 1 ore sample**

***Ore Type 2***

The Ore Type 2 sample was a 2-inch crush sample included in the analysis to serve as a second independent sample for model validation. Figure 7 presents the fraction of recoverable gold in each size class for the Ore Type 2 sample. Eighty percent of the Ore Type 2 gold is associated with ore sizes <0.5 inches.

Figure 8 presents model predicted and measured gold recovery for the Ore Type 2 sample. The model provides a good estimate of recovery (RMSE = 0.0355). The ability of the estimated parameters to more accurately predict gold recovery for the Ore Type 2 sample than the Ore Type 1 sample, may be due to the presence of ore sizes >2 inches in Ore Type 1 which resulted in lower than predicted recovery rates.

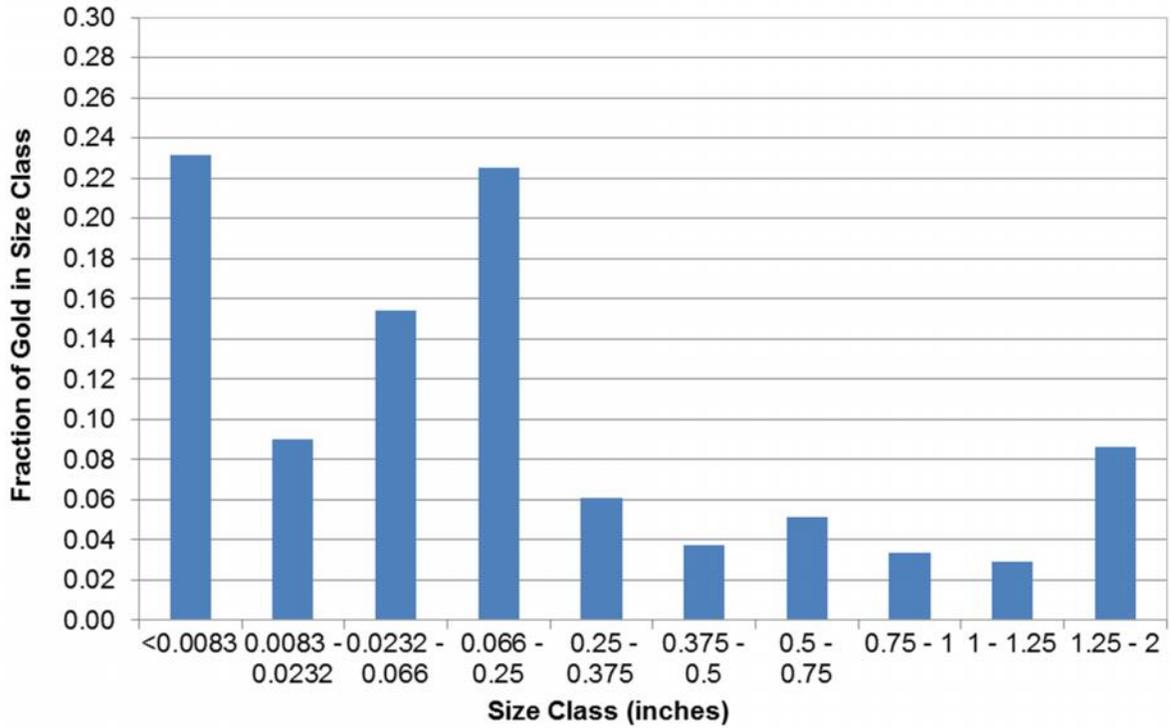


Figure 7. Gold distribution by particle size class for Ore Type 2 sample

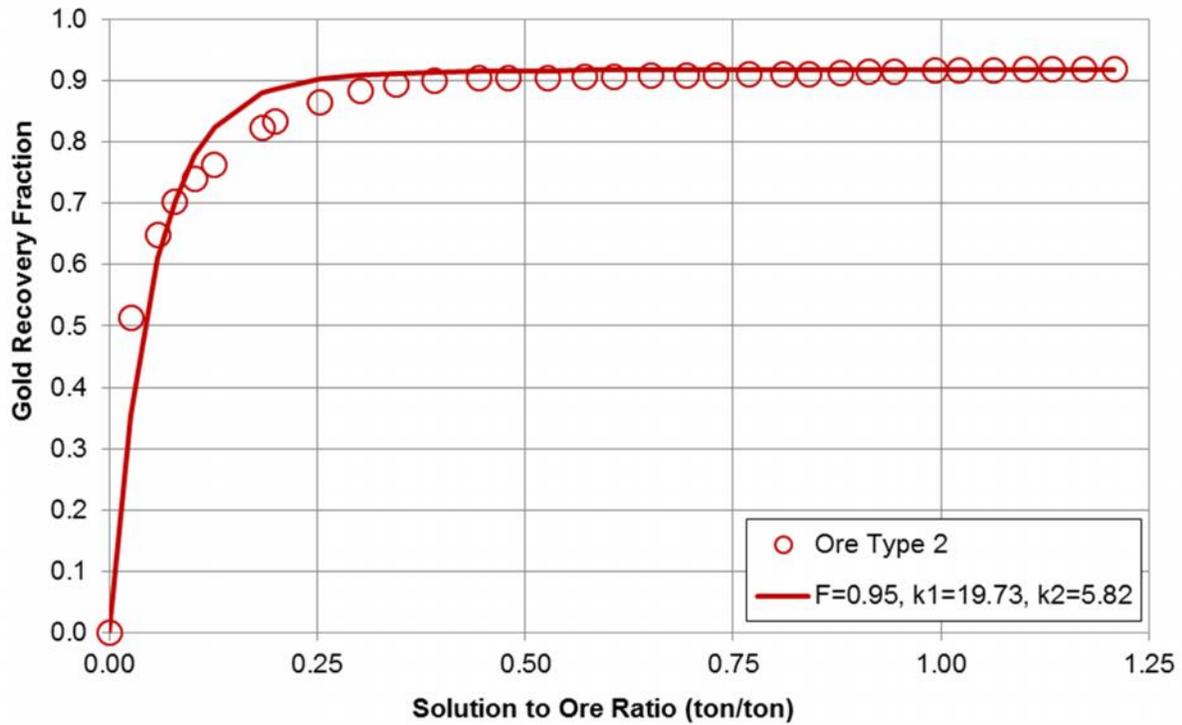


Figure 8. Model predicted and measured recovery for Ore Type 2 ore sample

## Heap Scale Model Predicted Recovery

The metal recovery model was applied to an operating heap leach pad to evaluate model performance and parameter values at the heap scale. The recovery model was initially calibrated to the measured recovery data from the first lift of the heap, which was comprised of two different ore types. Starting parameters for model calibration were derived from column data. The  $LE$  parameter (Equation 1) for the first lift was assumed to be equal to one. Solution to rock ratio data were estimated from leach pad ore stacking and irrigation records for the first lift.

The time delay for reporting of solution to the bottom of the heap was estimated from the estimated porewater velocity ( $v$ ) of the solution, calculated as:

$$v = \frac{I_i}{S} \frac{R}{F P} \quad (4)$$

The solution filled porosity was estimated from laboratory hydraulic property measurements and averaged  $0.27 \text{ cm}^3/\text{cm}^3$  for the range of operational irrigation rates.

The starting and best fit parameters for the two ore types are provided in Table 1. Figure 9 and Figure 10 present the measured and gold recovery model predicted recovered gold on a monthly basis and cumulated over the model period, respectively.

Ore 1 model parameters from the column data were adjusted downward to improve the match between model predicted and measured recovered gold for the first lift. Ore 2 model parameters remained unchanged.

**Table 1. Ore 1 starting and calibrated model parameters and Ore 2 model parameters**

Material	ERF	ERF <sub>&lt;0.5"</sub>	R <sub>max</sub>	F	k <sub>1</sub>	k <sub>2</sub>
Ore 1	0.90	0.53	0.81	0.90	7.20	4.50
Ore 1 Calibrated	0.65	0.30	0.68	0.65	3.76	1.22
Ore 2	0.71	0.53	0.66	0.71	8.07	1.66

Applying the calibrated model parameters to the second and third lifts required increasing the  $LE$  parameter to 1.05 for the second lift ore and 1.15 for the third lift ore to provide a more accurate prediction of cumulative recovered gold. The greater than one  $LE$  parameters for the second and third lifts indicates delayed recovery of gold from the underlying lifts. The degree of accuracy between model predicted and measured gold recovery was 0.4% for the model time period.

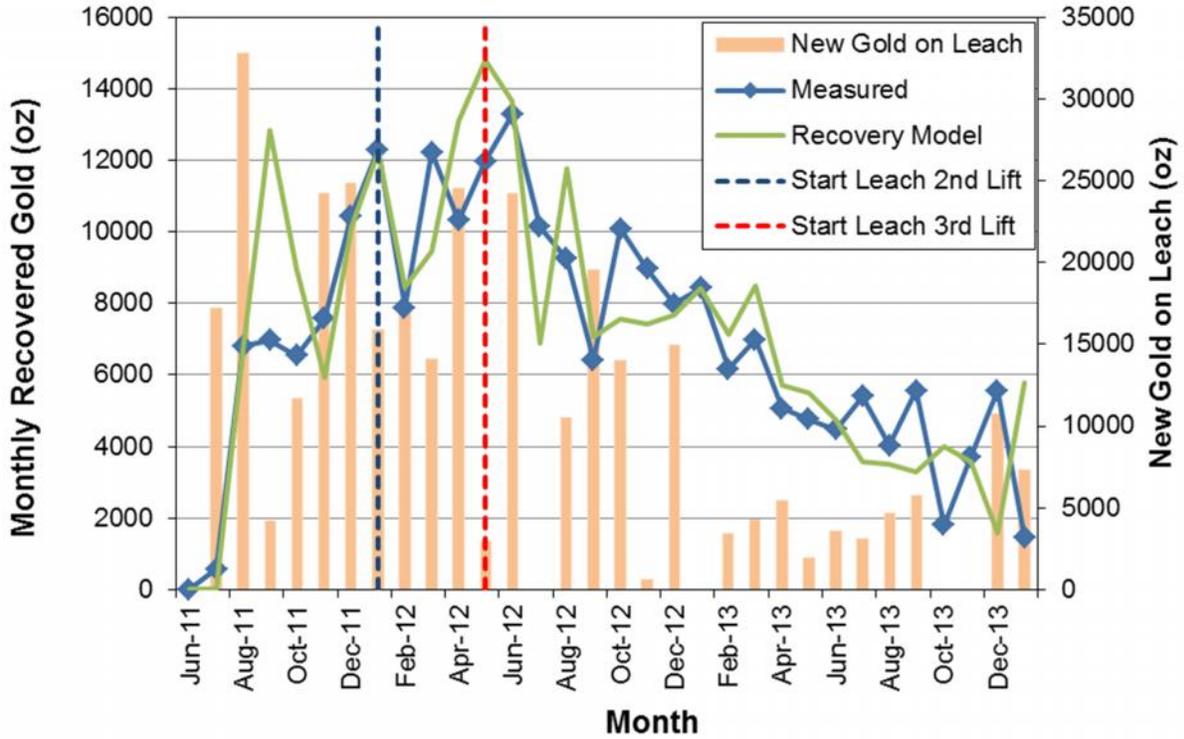


Figure 9. New gold under leach and measured and model predicted monthly gold recovery

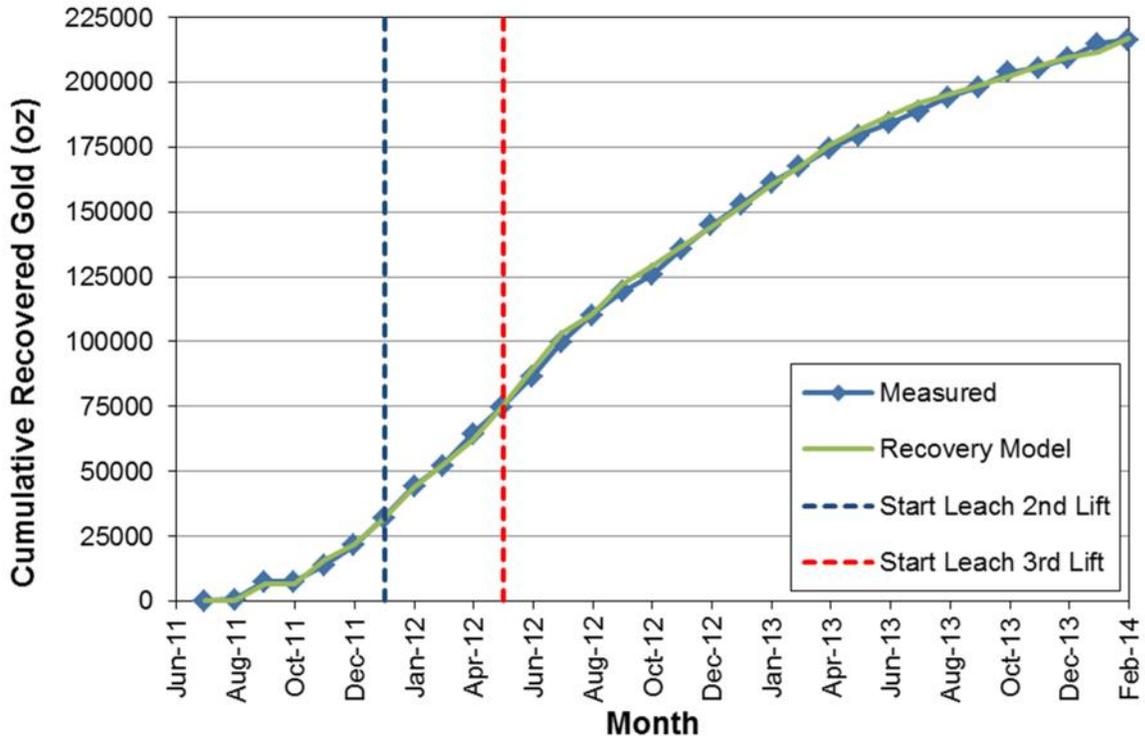


Figure 10. Measured and model predicted cumulative gold recovery

## Conclusions

An empirical metal recovery model that accounts for metal dissolution and transport in two domains has been developed. The model uses a physically based, ERF algorithm to estimate the model parameters. Estimated parameters were applied to predict column and heap leach scale recovery data and results indicates the ERF algorithm provides a means to adjust parameter estimates which are able to adequately simulate variations in recovery that may be due to permeability or metallurgical constraints that may inhibit metal dissolution or movement. Additional improvements in the ERF algorithm may be realized by expanding the correlation dataset to include additional samples with different particle size distributions and metallurgical compositions.

The calibrated recovery model provided very good agreement between heap leach scale predicted and measured gold recovery. This indicates that the recovery model calibrated to field data can accurately predict gold recovery of subsequent lifts with slight adjustments to the *LE* parameter only. The ability to accurately predict gold recovery with the adjustment of a single, physically based parameter allows for employing the model to better understand gold recovery limitation. For example, the model may be used to indicate if longer leach times (or shorter lifts) are needed to reach recovery goals.

## Acknowledgements

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