New methods for hydraulic characterization of mine waste and cover system materials

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Abstract

Successful cover system design requires an accurate understanding of the hydraulic properties of both the cover material and underlying mine waste. Mine waste and frequently available borrow material for cover systems may contain a significant fraction of gravel material (>4.75 mm) which influence the material hydraulic properties. Over the last decade, we have developed several laboratory methods to quantify unsaturated flow behaviour in gravelly materials. These consist of using large diameter cores, directly measuring the hydraulic conductivity function at near saturation, and using flexible wall methods to minimize wall effects. Results from characterization work on waste rock and heap leach samples indicate that 1) unsaturated flow parameters derived from material without gravel cannot represent the actual hydraulic behaviour; 2) basing the hydraulic conductivity function on saturated hydraulic conductivity cannot describe measured unsaturated flow data; 3) a dual-permeability model can improve predictions of flow behaviour, and; 4) a flex-wall column design allows for successive measurements of hydraulic conductivity and air permeability under variable bulk density conditions evaluated during the cover system design phase.

1 Introduction

Successful cover system design requires an accurate understanding of the hydraulic properties (e.g. hydraulic conductivity, soil water retention capacity) of both the cover material and underlying mine waste. Mine waste and frequently available borrow material for cover systems may contain a significant fraction of gravel material (>4.75 mm) which influence the material hydraulic properties. Previous work has shown that the practice of removing gravels to facilitate the use of standard laboratory hydraulic characterization methods produces highly inaccurate hydraulic property results. Nonetheless, including gravels in hydraulic characterization methods presents several challenges: large diameter flow cell are needed to minimize gravel particle interactions; solution and air flow along the side-walls may bias results due to non-uniform contact between the material and the column, and; the influence of macropores on flow may invalidate assumptions for predicting the hydraulic conductivity function of soil materials.

Over the last decade, we have developed several laboratory methods to quantify unsaturated flow behaviour in gravelly materials. These consist of using large diameter cores, directly measuring the hydraulic conductivity function at near saturation, and using flexible wall methods to minimize wall effects. Results from characterization work on the waste rock and heap leach samples from the southwest United States, Chile, and Peru indicate that 1) unsaturated flow parameters derived from material without gravel cannot represent the actual hydraulic behaviour; 2) basing the hydraulic conductivity function on saturated hydraulic conductivity cannot describe measured unsaturated flow data; 3) a dual-permeability model can improve predictions of flow behaviour, and; 4) a flex-wall column design allows for successive measurements of hydraulic conductivity and air permeability under variable bulk density conditions evaluated during the cover system design phase.

2 Background

The presence of gravel particles (> 4.75 mm diameter) can significantly affect the moisture retention characteristic (MRC) and unsaturated hydraulic conductivity function (K(θ)) of soils. In particular, the typically gravelly nature of waste rock and heap leach material creates large macro-pores that can significantly affect flow and transport behaviour of both solution and air within these materials. Some researchers have shown that increasing gravel content decreases the saturated hydraulic conductivity (K_s) (Bouwer and Rice, 1984; Dunn and Mehuys, 1984), whereas others have shown that K_s can increase or decrease depending on the percentage of gravel fragments (Milczarek et al., 2006; Cousins et al., 2003; Poesen and Lavee, 1994). The water holding capacity of the gravelly soils tends to be smaller than that for the nongravelly soils at matric potential heads less than -100 cm (Milczarek, 2006; Khaleel and Heller, 2003). Air entry values have also been observed to decrease as gravel contents exceed 30 percent (Milczarek, 2006).

Typically, MRC and K_s data are obtained through laboratory experiments on the fine fraction (< 2 or 4.75 mm grain size) of the field-collected samples. These data sometimes are used directly without considering gravel effects. In other cases, correction factors are applied to the measured data to account for the effect of gravel on the MRC and K_s based on the volumetric or gravimetric gravel content. Several researchers have shown however (Milczarek et al., 2006; Cousin et al., 2003; Khaleel and Heller, 2003), that a simple correction factor can lead to highly erroneous estimates of the MRC relation and K_s.

For soils with a high percentage of gravel particles, the spatial distribution of the gravel can create macro-pores and discontinuity in the pore size distribution. Two or more distinct regions in MRC have been observed by several researchers (Milczarek et al., 2006; Al-Yahyai et al., 2006; Zhang and Chen, 2005; Poulsen, 2002). Therefore, the MRC and K(θ) for the gravelly soils should require two or more functions to describe the unsaturated hydraulic properties for the entire pore-size distribution. Larsbo et al. (2005) proposed a dual-permeability model MACRO 5.0, in which the overall pore space is divided into macropores and soil matrix. A kinematic wave equation (Germann, 1985) is used to describe water flow in macropores, while the Richards' equation is used to simulate water flow in soil matrix. Most unsaturated flow models (e.g. HYDRUS, UNSATH, SWIM, and SoilCover) are limited to Richards' equation to describe water flow, however, the existence of macropores can lead to preferential flow which cannot be simulated by the Richards' equation.

The ability to accurately characterize and model gravelly materials is vitally important because we build models to design for mine closure (e.g. estimate heap leach draindown, cover performance in limiting deep percolation, and to estimate water balance). Incorrect model estimates can be costly, leading to oversized or undersized water treatment systems, excess deep percolation, under predicted water holding capacity of waste, and under predict drainage response to storm events. Thus, the need is supported for cost-effective methods to characterize gravelly materials that will be representative of the material and truly define unsaturated flow characteristics.

Measurement of soil water retention and unsaturated hydraulic conductivity of gravelly materials are difficult because different measurement methods are needed for near saturation, moderate tensions, and dry tensions due in part to large changes in flow with small changes in water content at the wet range and extremely slow water movement in moderate to dry tension range potentially making for unreasonably long test times. Cost and sample size is an important consideration; large sample sizes are needed for representativeness, however large columns generally equate to increased analysis time and material costs.

In this paper, laboratory experiments were conducted for a gravelly soil to determine the MRC and K_s with the gravel included (bulk sample) and with particles > 4.75 mm diameter removed. In addition, direct K_{unsat} measurements were conducted at three known unsaturated flow rates for better prediction of the K(θ) relation. Unsaturated flow was then simulated using the computer codes MACRO 5.0 and HYDRUS-1D to compare the best fitting methods and predictive results. Finally, we present results from an innovated flexible wall column design that allows for successive measurements of material hydraulic properties under variable bulk density conditions.

3 Methodology

The following briefly describes the gravelly material hydraulic testing procedures implemented in our lab and utilized in this study.

3.1 Hydraulic Testing Methods for Gravelly Material

Particle size analysis is first performed on the gravelly sample to determine the column size to be utilized for analysis. Sizing follows standard ASTM procedure (ASTM, 2000) which recommends that column diameter be a minimum of eight times the largest particle diameter. Due to size logistics associated with extremely large gravels (e.g. sample size required, equipment size constraints for the laboratory), material greater than 19 mm (0.75 inch) is removed, with the limitation that no more than 20% of the sample (by mass) is removed. Removal of 20% or less of the large gravel fraction has been shown to not significantly influence hydraulic behaviour (Milczarek et al., 2006). If removing all material greater than 19 mm (0.75 inch) surpasses the 20% threshold, then the particle size ceiling is raised to 38 mm (1.5 inch) and the column diameter is increased to meet the 8X column diameter standard. For example, 38 mm (1.5 inch) maximum particle diameter would use a 30 cm (12 inch) diameter column.

The column is instrumented with water content and tensiometer sensors that are continuously logged using an attached datalogger. A 1 bar air-entry porous place is fit to the bottom of the column to allow for measurement of soil water retention characteristics. Soil water characteristics is measured using hanging column and tempe cell methods (Dane and Hoppmans, 2002) for wet and moderate tensions, respectively. Dry point retention is measured outside the column using the chilled mirror method (Gee et al., 1992). Following MRC analysis the porous plate is removed and the column is then utilized to measure K_{unsat} using the long-column approach (Corey, 2002) and K_s using the constant head method (Reynolds and Elrick, 2002). For K_{unsat} measurements, solution is applied to the surface of the material through an evenly distributed network of irrigation points at rates of 10^{-3}, 10^{-4}, and 10^{-5} cm/sec with an approximate matric potential of -10 cm applied to the bottom of the core. Figure 1 shows the typical experimental setup during K_{unsat} testing.

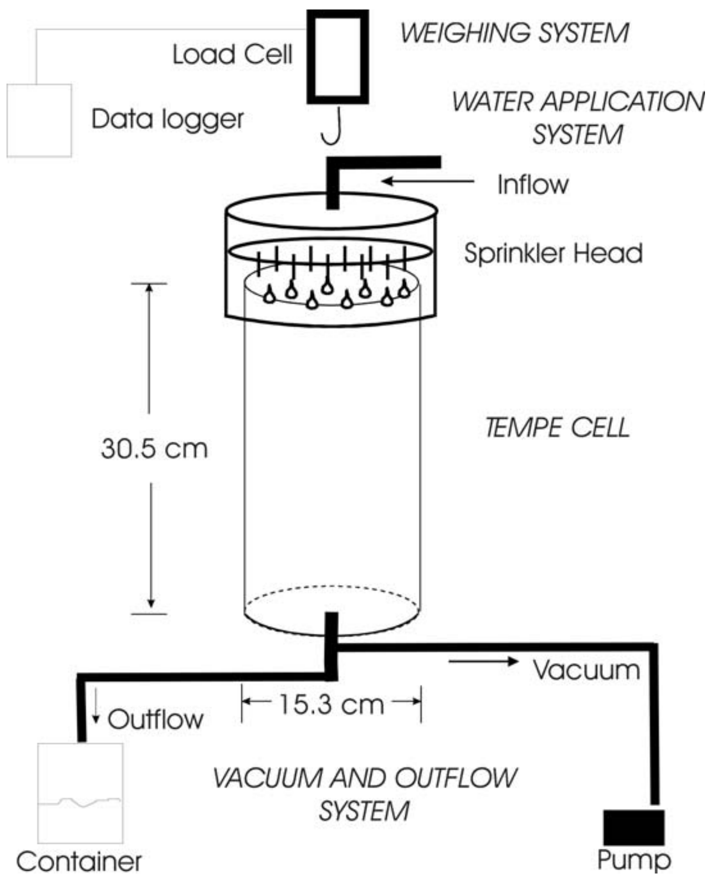

Figure 1 Standard large column setup for unsaturated hydraulic conductivity testing

3.2 Bulk Sample and < 4.75 mm Sample Preparation and Testing

Laboratory experiments were conducted for a gravelly waste rock sample to determine the MRC and K_s with the gravel included (bulk sample) and with particles > 4.75 mm diameter removed. The particle size distribution of the bulk and < 4.75 mm sample is presented in Figure 2.

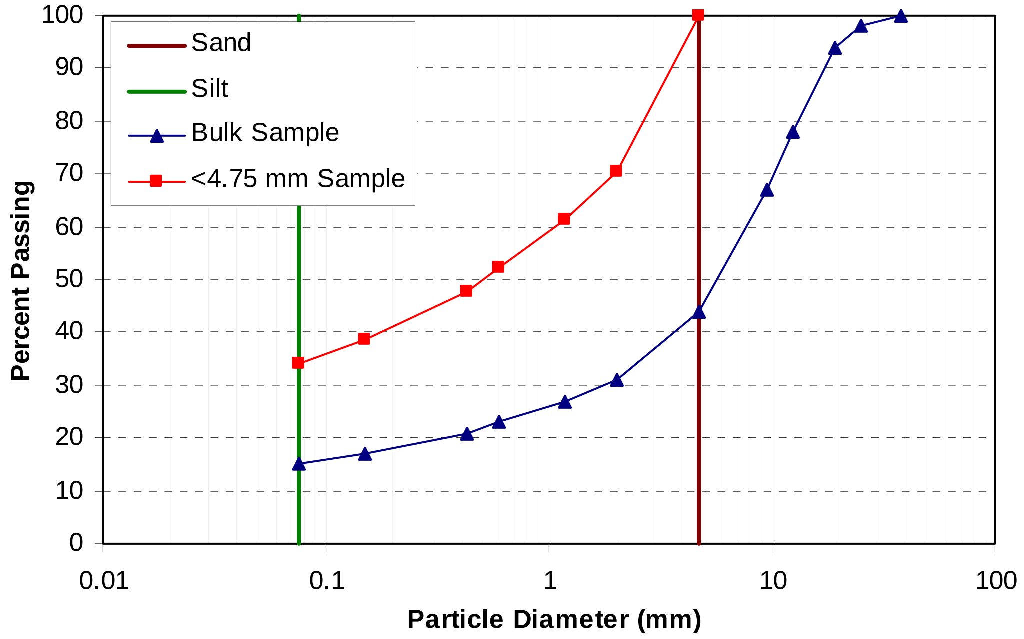

Figure 2 Particle size distribution for waste rock sample with gravel fraction (bulk sample) and with particles > 4.75 mm removed

Characterization of the bulk sample was done in a 30 cm diameter by 30 cm long column following the procedure outlined in the preceding section. The < 4.75 mm particle diameter sample was obtained by sieving the bulk sample to remove particles > 4.75 mm in diameter. This samples was then packed at a bulk density of approximately 1.65 g/cm^3 into a 5 cm diameter by 18 cm long core for MRC and K_s measurements using the tempe cell and constant head methods, respectively. The experimental procedure utilized in this study is summarized in Figure 3.

3.3 MRC and K_{unsat} Fitting/Estimation

MRC and K(θ) functions were developed from the measured MRC, K_s, and K_{unsat} data via three methods.

Method 1 - The standard van Genuchten (SVG) equation (van Genuchten, 1980) was fitted to the MRC data for the total range of pore sizes. The corresponding standard van-Genuchten-Mualem (SVGM) K(θ) relation (van Genuchten, 1980) was estimated using the optimized MRC fitting parameters, measured K_s, and a fixed shape factor, L, of 0.5.

Method 2 - The measured MRC on the < 4.75 mm diameter sample material was used to parameterize the SVG equation and the K(θ) relation was estimated from the SVGM model with measured K_s and L equal to 0.5.

Method 3 - The modified van Genuchten (MVG) equation of Vogel (2001) was fit to the MRC data excluding data at or close to saturation (> -5 cm matric potential head) to eliminate macropore flow effects. Parameters of the modified van Genuchten-Mualem (MVGM) equation by Luckner (1989) were then determined by fitting to the measured K_{unsat} data, while fixing the van Genuchten parameters to the previously determined values. The hydraulic conductivity function in macropores is given as a simple power law of the macropore degree of saturation, S_{ma}:

$$K_{ma} = K_{s(ma)} S_{ma}{}^{n^*}, \ \theta > \theta_b \qquad (1)$$

where $K_{s(ma)}$ is the saturated hydraulic conductivity of the macropores, and n^* is a "kinematic" exponent reflecting macropore size distribution and tortuosity.

The computer program RETC4 (van Genuchten et al., 1991) was used to fit the experimental data using methods 1 and 2. For method 3 the MRC function was fitted in RETC4 while fitting of the K(θ) function was conducted with a spreadsheet model.

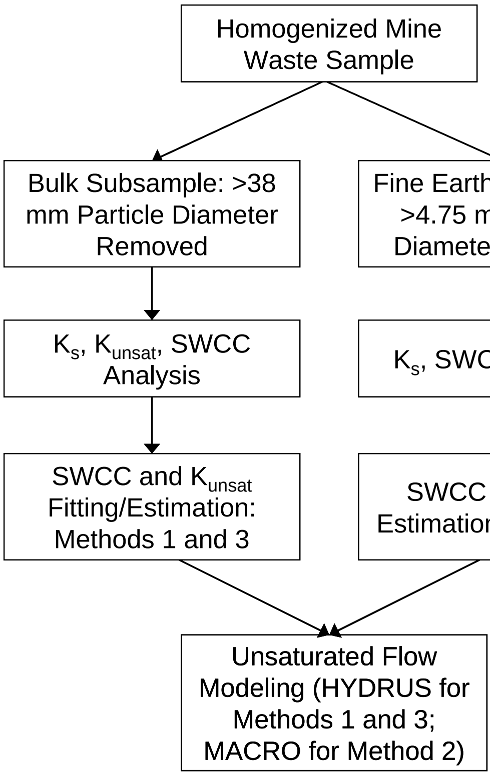

Figure 3 Experimental procedure

3.4 Unsaturated Flow Modeling

Unsaturated column outflow experiments at the 2×10^{-4} cm/s irrigation rate were simulated with computer codes HYDRUS-1D (Simunek et al., 1998) and MACRO 5.0 (Larsbo et al., 2005). Parameters obtained using MRC and K(θ) fittings/estimation methods 1 and 2 were used to parameterize HYDRUS-1D. HYDRUS-1D does not have an option to implement the MVGM equation, thus unsaturated flow modeling using MRC and K(θ) functions parameters from method 3 were done using MACRO 5.0 (Larsbo et al., 2005).

A 30 cm long one dimensional model domain discretized into 200 nodes was defined. Initial conditions were defined based on the observed water contents. During irrigation a constant flux top boundary condition of 2×10^{-4} cm/s was assigned; during drainage the top boundary condition was zero flux. The lower boundary condition was set as free drainage.

4 Data and Results

4.1 MRC and K(θ) Results

Figure 4 shows the measured MRC data and the fitted MRC functions using Methods 1-3. Table 1 lists the fitted SVGM and MVG parameters and the measured K_s. The MRC measured data for samples with and without gravel particles deviate substantially from one another, indicating a significant influence from gravel content on the MRC. It should be noted that measured water content values did not appear to reach equilibrium in the bulk sample at matric potentials less than -10 cm, possibly due to low hydraulic conductivity and reduced pore connectivity. The fitted MRC function using method 3 (MVG) show no significant difference from Method 1 at intermediate and low matric potential heads, but differ at high (near zero) heads. This result is expected given that the MVG method excludes matric potential data from near saturated conditions and assumes a lower saturated water content.

Table 1 Fitted van Genuchten-Mualem and modified van Genuchten-Mualem parameters and measured K_s

Method	Saturated Hydraulic Conductivity	Unsaturated Hydraulic Parameters (van Genuchten parameters)				
		Residual Water Content	Saturated Water Content	a	N	L
	(cm/s)	(cm^3/cm^3)	(cm^3/cm^3)	(cm^{-1})	(-)	(-)
1 - SVGM	>5.6E-2[a]	0.000	0.422	0.565	1.238	0.500[b]
2 - < 4.75 mm SVGM	>3.5E-02[a]	0.003	0.378	0.012	1.190	0.500[b]
3 - MVGM[c]	2.10E-04	0.000	0.312	0.314	1.267	7.217

[a]Beyond apparatus measurement capacity; [b]Fixed value; [c]Parameters for soil matrix in Vogel (2001) function

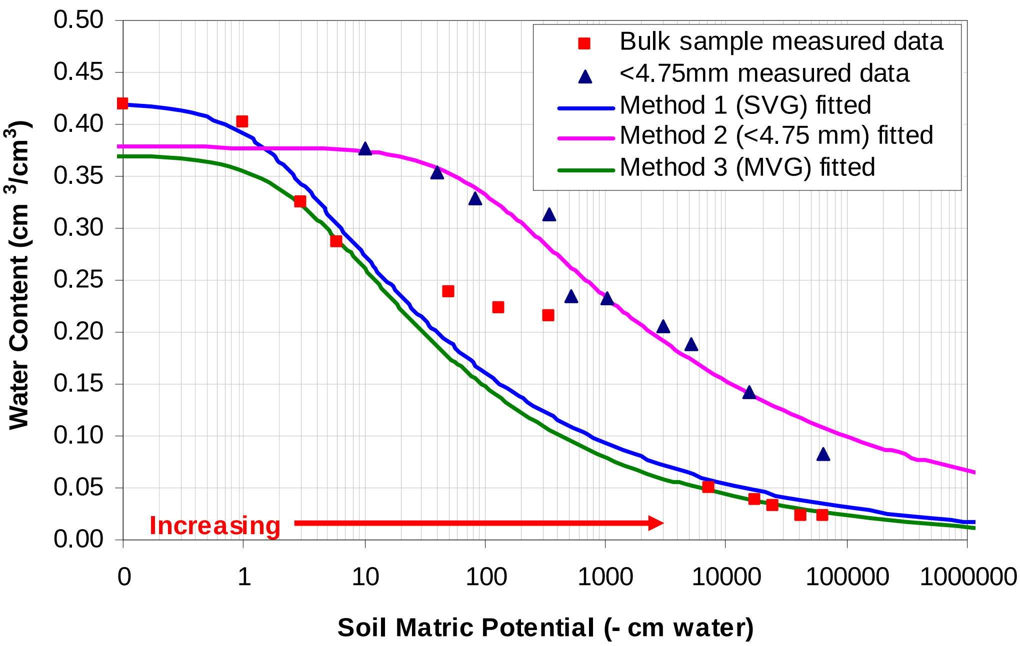

Figure 4 Measured and fitted MRC

Figure 5 shows measured, estimated and fitted K(θ) relations. The estimated K(θ) using Method 1 (SVGM) and Method 2 (<4.75 mm SVGM) reasonably approximated the measured K_{unsat} at the lower water content, but both diverge from the measured data as the water content increases. Method 3 (MVGM) showed the best agreement with the measured K_{unsat} data under the assumption that macro-pores and the soil matrix partition correspond to a water content of 0.31 cm^3/cm^3 and soil matric potential head of -3 cm.

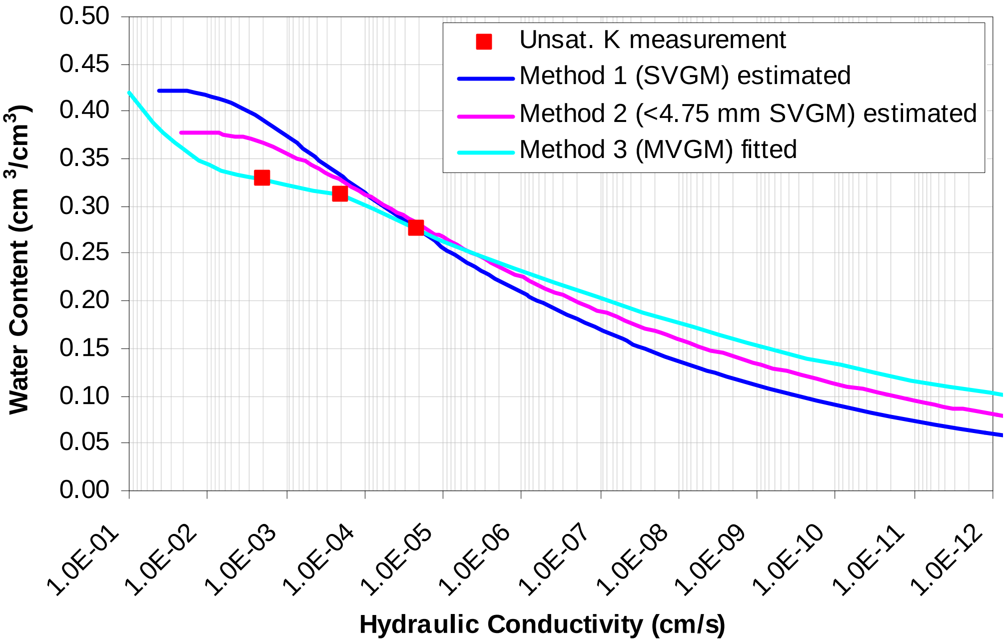

Figure 5 Measured and estimated or fitted K(θ)

4.2 Unsaturated Flow Simulation Results

Figure 6 shows the observed water contents during the unsaturated flow experiment and the HYDRUS-1D and MACRO 5.0 model simulated water contents using corresponding parameters obtained by Methods 1 through 3. Additional input parameters required for implementing Method 3 (MVGM) using the MACRO 5.0 model were a total K_s and porosity (both macropores and soil matrix) of 0.1 cm/s and 0.42, respectively, and n* (equation 1) of 2.0.

In general, the simulations accurately predicted the wetting front arrival time and the initial drainage time, however, the predicted water content during irrigation and drainage rates varied considerably. The Method 1 (SVGM) simulation overestimated water content at unit gradient conditions and also overestimated the drainage rate. That is, the simulations predicted a faster decrease in water content during the drainage phase than observed. The method 2 (SVGM parameters from < 4.75 mm material) simulation over estimated water content and drainage rate. Method 3 (MVGM) simulation results were in better agreement with the observed water contents than the results from Methods 1 and 2 simulations, however, similar to the other methods it over predicted drainage rate.

To assess the accuracy of the methods in modeling the matric potential heads during irrigation and drainage, the head data collected at 10 cm from the top of the column during the column outflow experiment was compared against simulated results using Methods 1 through 3. Figure 7 shows that the simulated matric potential heads for methods 1 and 3 do not agree with observed data during the transient flow periods, but reasonably predict matric potential heads during steady-state conditions. Most noteworthy were the simulated matric potential heads using Method 2 (< 4.75 mm SVGM parameters) which differed from the measured data by 3 to 4 orders of magnitude (data not shown). This behaviour is explained by the hugely contrasting measured MRC data between the bulk sample and sample without gravel particles (Figure 4).

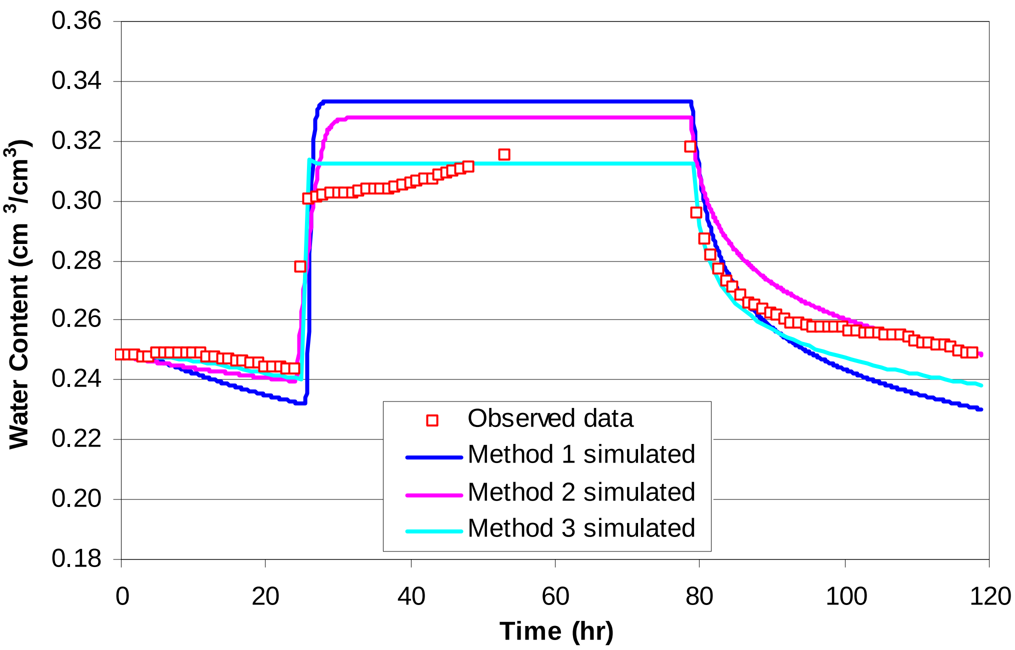

Figure 6 **Measured water content during irrigation experiment and simulated values using parameters from Methods 1 through 3**

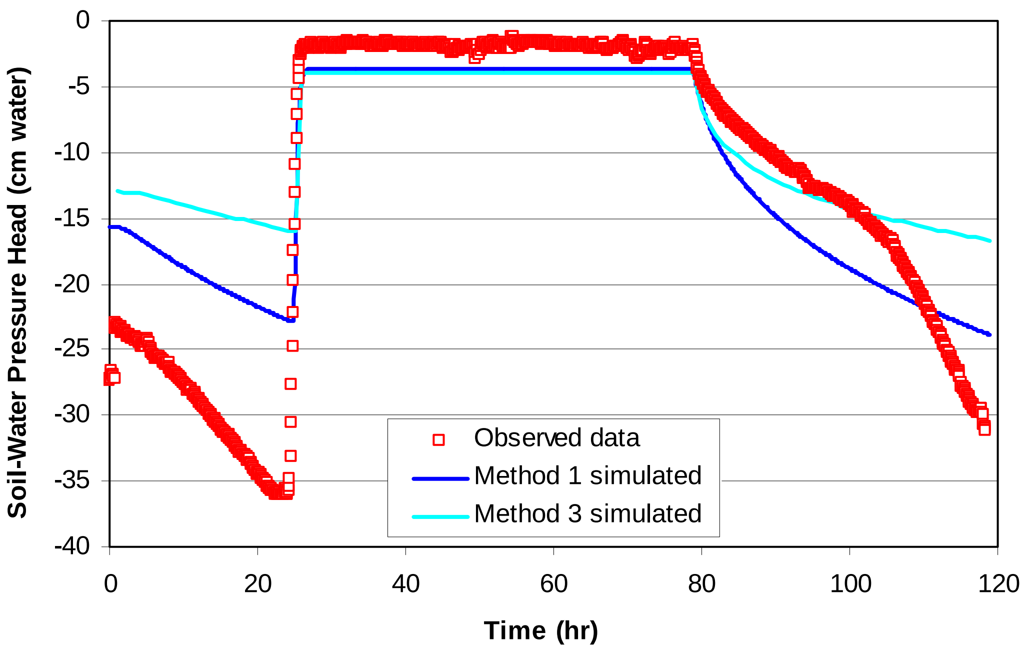

Figure 7 **Measured soil-water pressure heads during irrigation experiment and simulated values using parameters from Methods 1 and 3**

Table 2 lists the performance of the different methods used to fit the MRC data and fit or estimate the K(θ) relations, as well as the performance of the parameters when used in simulations of unsaturated flow. The performance levels (i.e., poor, average, better, best) are relatively defined according to the calculation results of the root-mean-square error (RMSE). Among the three procedures, Method 3 (MVGM) showed the best performance for representing the MRC and K(θ) relations as well as simulating the outflow experiments with MACRO 5.0. Methods 2, which relied on MRC measurements using the $<$ 4.75 mm size fraction and gravel corrected parameters, generated the worst performance. In representing the K(θ) relation and simulating water content during the outflow experiments, Method 2 (< 4.75 mm SVGM) generally outperformed Methods 1 which relied on the bulk sample. However, Method 2 performed significantly worse than Methods 1 and 3 in simulating observed matric potential during the outflow experiment. The failure of agreement between the observed and predicted data from Method 2 indicates that removing the gravel fraction from gravelly soil material results in unreliable representations of the MRC, K(θ) and K(h) functions. Additionally, the improved agreement between the observed and predicted data from Method 3 indicates that a dual-permeability model is needed to accurately represent flow characteristics in gravelly material.

Table 2 Relative performance of each method in MRC and K(θ) fitting or estimation and unsaturated flow modeling. Associated RMSE are in parenthesis.

Method	MRC Fitting	K(θ) Fitting/Estimation	Water Content Simulation	Pressure Head Simulation
1 - SVGM	Average (0.037)	Average (0.639)	Average (0.021)	Average (6.202)
2 - <4.75 mm SVGM	Poor (0.088)	Better (0.581)	Better (0.012)	Poor (>100)
3 - MVGM	Average (0.047)	Best (0.049)	Better (0.013)	Average (8.745)

5 Flex Wall Measurements

GeoSystems Analysis, Inc. has developed an innovated flexible wall column design that allows for waste rock and ore material to be consolidated in place within the column by an inflatable side-wall. The purpose of developing the flexible wall testing device are to: 1) minimize the edge effect between test material and rigid wall; 2) to enable different hydraulic tests at variable bulk densities in one flow column, thus less sample is needed; 3) to reach a more uniform compaction and avoid vertical differential compaction that occurs with traditional vertical compression tests.

Presented are two examples of ore material with different particle size distributions (Figure 8). Ore 1 is a well sorted ore with 59 and 19 percent passing the #4 and #200 mesh, respectively. Ore 2 is poorly sorted with 34 and 4 percent passing the #4 and #200 mesh, respectively. The two ores were packed into 15 cm (6 inch) diameter by 30 cm (12 inch) long flexible side-wall columns and measurements made of bulk density, K_s, and air permeability as a function of side-wall pressure. K_s was measured using the long-column (Corey, 2002) method and air permeability was measured using the steady-state method (Ball and Schjonning, 2002).

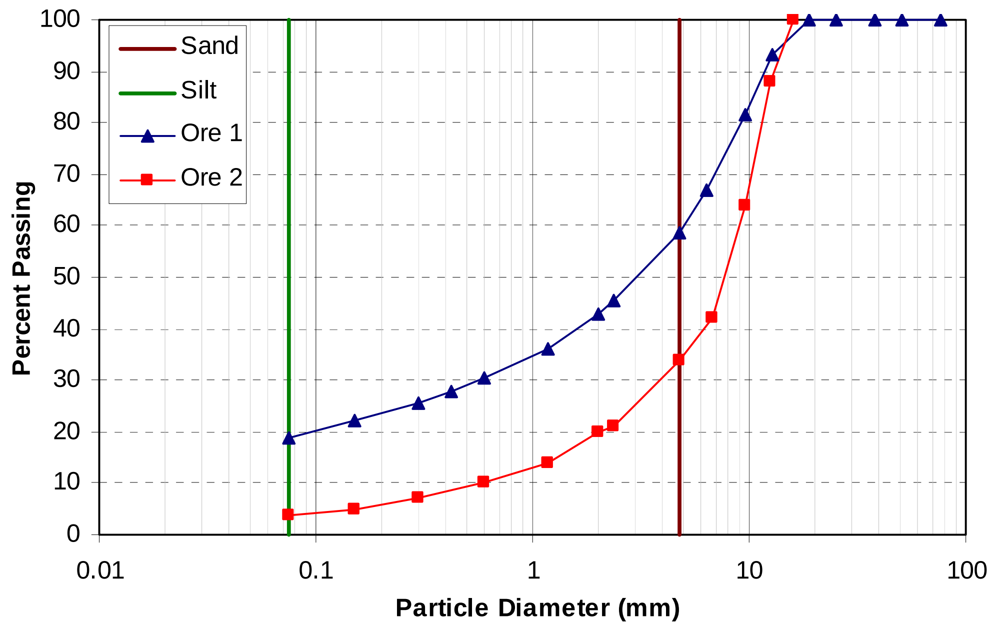

Figure 8 Flexible wall column ore samples particle size distribution

The ability of the flexible side-wall column to differentially increase bulk density based on material characteristics is shown in Figure 9. As would be expected, the poorly sorted ore (Ore 2) is more easily compressed than the well sorted ore (Ore 1) as small particles fill in the voids created by the gravel matrix. Figure 10 shows the increase in K_s for the two ores as the wall pressure is increased. A more steep decline in K_s for the poorly sorted ore (Ore 2) compared to Ore 1 is observed and corresponds with the observed compressibility of the two ores (Figure 8).

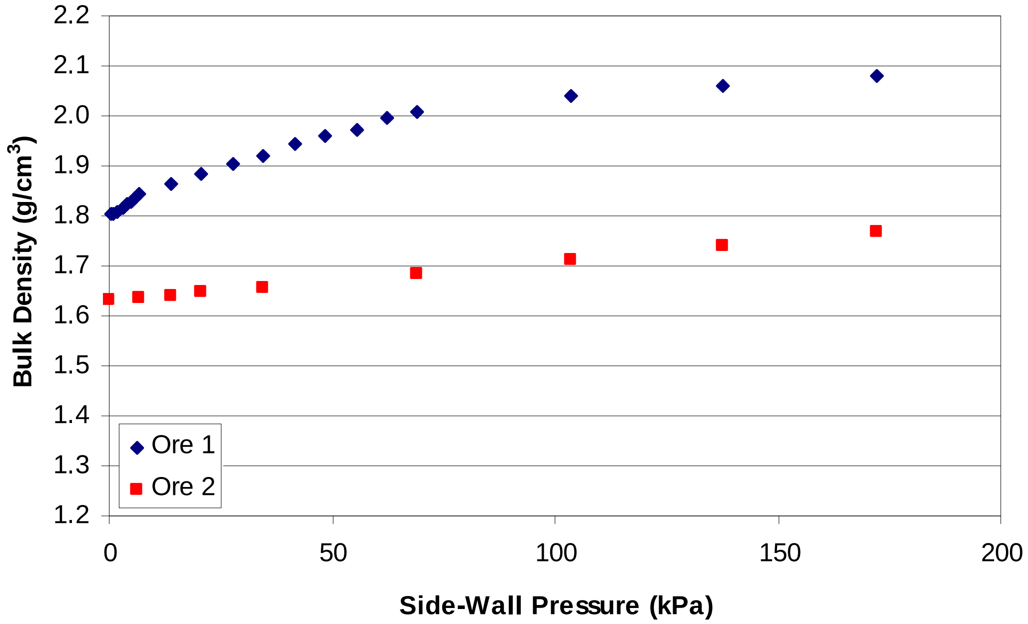

Figure 9 Flexible wall column measured ore bulk density as a function of side-wall pressure

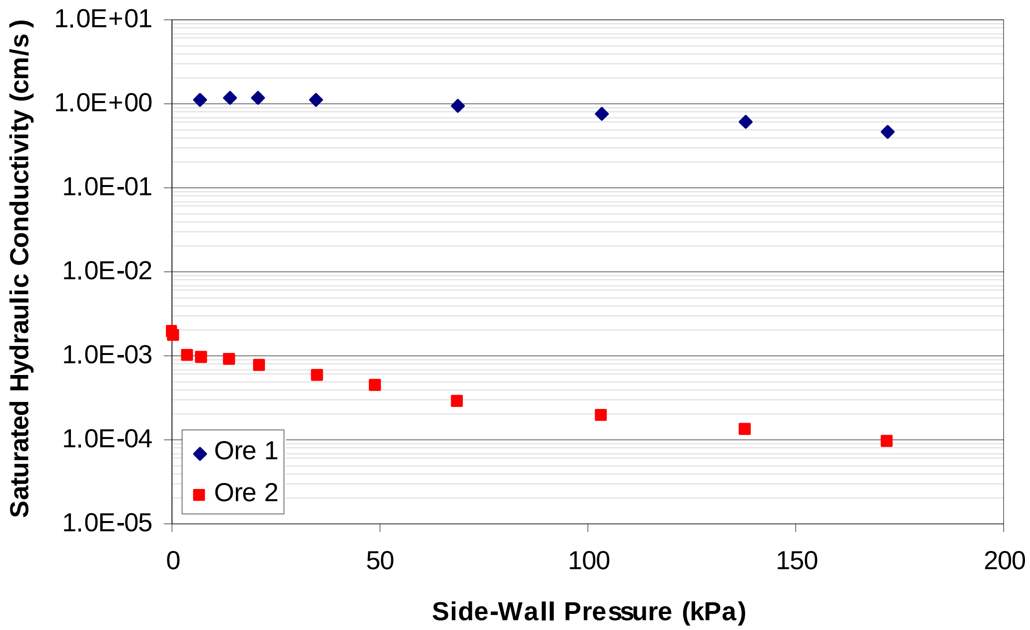

Figure 10 Flexible wall column measured ore saturated hydraulic conductivity as a function of side-wall pressure

Air permeability for Ore 1 and Ore 2 are presented in Figure 11. At 50% air saturation, air permeability for Ore 1 at a side-wall pressure of approximately 14 kPa (bulk density of 1.86 g/cm^3) is an order of magnitude less than air permeability for Ore 2, which were analysed at side-wall pressures of 0, 68, and 172 kPa (bulk density of 1.56, 1.67, and 1.78 g/cm^3). The reduced air permeability of Ore 1 at a lesser side-wall pressure than Ore 2 is attributed to Ore 1 being highly compressible relative to Ore 2. Ore 2 air permeability does not change significantly with increasing bulk density (i.e. increasing side-wall pressure) as a result of its low compressible character.

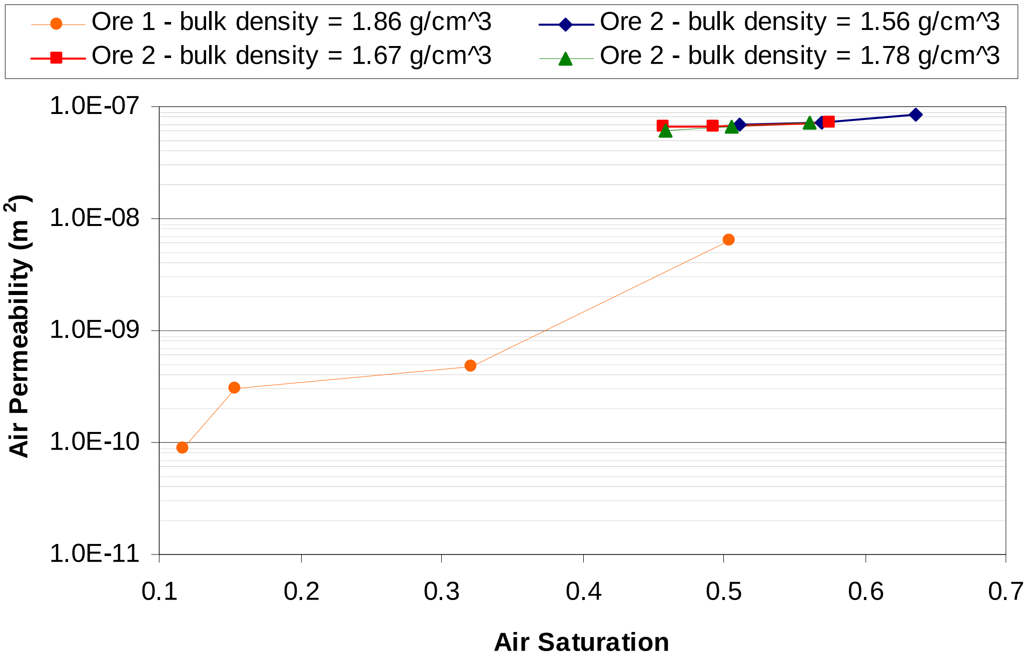

Figure 11 Flexible wall column measured ore air permeability function at different bulk densities (i.e. side-wall pressures)

While not currently supported by present data, it is our belief that the flexible-wall column reduces or eliminates side-wall air flow that is prevalent in rigid wall columns during air permeability testing on coarse textured materials. The flexible side-wall column is believed to alleviate side-wall flow by way of its ability to mold around jagged edges of the gravel material. Laboratory work is ongoing to evaluate the influence of the flexible side-wall column in cutting off side-wall flow and to compare flexible side-wall column results to the more traditional load permeability tests.

6 Conclusions

Direct unsaturated flow and MRC experiments with gravelly waste rock material packed into a large diameter column showed a significant difference in unsaturated flow and MRC properties in the same material where gravel had been removed. A modified van Genuchten-Mualem equation, which discretizes pore space into matrix and macropore flow, best represented the measured K(θ) relation and observed outflow experiment conditions for a gravelly soil. Predictions based on van Genuchten parameters derived from soil with the gravel removed or that did not utilize a dual-permeability model substantially deviated from the measured K(θ) and observed outflow data. These results indicate that accurate characterization of gravelly material requires inclusion of the gravel during hydraulic property measurements and that direct measurement of K_{unsat} along with the use of an appropriate duel-permeability model can more accurately describe the behaviour of flow in gravelly waste rock and heap leach material. Furthermore, the application of an innovative flexible side-wall column design allows for successive measurements of hydraulic properties under varying pressures, providing for the ability to more easily analyse waste rock and leach ore hydraulic characteristics under varying bulk density.

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